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Mathematics Textbooks for Prospective Elementary Teachers: What's in the books?

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Abstract

This paper reports on an analysis of mathematics textbooks written for use in courses designed for prospective elementary teachers. We address two questions: 1) How do the contents of these books compare overall? 2) What are similarities and differences across the textbooks in three specific topics – fractions, multiplication, and reasoning and proof? In this study, we find that book content is consistent at the level of chapter titles and topics included: the books "cover" the same material. We find, however, that the level of detail, depth and breadth of approaches, presentation of material, and functionality of the books varies widely. With respect to the selected topics, the books vary in how they introduce the topics, what they include, and how they connect mathematics within and across topics.

Mathematics Textbooks for Prospective Elementary Teachers: What's in the books?

The importance of textbooks

Textbooks are often the primary source of teaching material for K-12 classroom teachers (Sosniak & Perlman, 1990). Some argue that textbooks create a national curriculum for mathematics and science in K-12 schools. At the same time, we know that in many cases, teachers use textbooks flexibly, changing the order of topics, picking and choosing what they teach (Freeman, 1983; Schmidt *et al.*, 1997; Stake & Easley, 1978; Stodolsky, 1988, 1989). While the extent of the influence of textbooks and other curriculum materials has been the subject of research and debate, it is undisputed that textbooks have a substantial impact on both what is taught and how it is taught in K-12 schools. At a minimum, textbooks are influential in determining what students have an opportunity to learn in K-12 mathematics (Porter, 1988).

We know less about how textbooks are used in mathematics courses at the undergraduate level. Do college instructors similarly depend on textbooks? In particular, there is little evidence about how textbooks are used in mathematics classes that prospective elementary teachers are required to take across the United States. Data from the Conference Board of the Mathematical Sciences (CBMS) survey of mathematics departments (Lutzer *et al.*, 2002) indicates that institutions have different approaches to mathematics classes for prospective elementary teachers: In the 84% of four-year institutions that certify elementary teachers, 77% offer a course or course sequence specifically designed for prospective elementary teachers; 7% designate special sections of other courses; while the remainder expect those students to meet a mathematics requirement in other ways. In two-year colleges involved with teacher preparation, 49% offer special mathematics classes for elementary teachers and 15% designate sections of other classes. Overall, whether in special classes or regular mathematics classes, 45% of four year colleges offering certification require two courses for early elementary teachers (grades K-3) with others varying from no required courses (8%) to five or more (6%). Although the CBMS report *The Mathematical Education of Teachers* (Conference Board of the Mathematical Sciences, 2001) calls for a minimum of 9 semester hours (3 courses) for early elementary teachers and up to 21 hours (7 courses) for later grades' teachers, the actual numbers for later grades suggest that this recommendation has not been widely adopted: 42% require 2 courses, 7% require none, and 18% require 5 or more (pp. 51-54).

These courses enroll large numbers of students across the country: the CBMS estimate in 2000 was 68,000 students enrolled in special mathematics courses for elementary teachers at four-year institutions, and an additional 16,900 at two year institutions. Thus, textbooks written for this audience potentially reach nearly 84,000 students each year.

Do these courses use textbooks? Anecdotal evidence suggests that they do. There are 14 such books currently in print, with others in preparation by mathematicians or mathematics educators who teach such courses. Of these 14 textbooks, seven are in their 3rd or higher edition, with one in 8th edition (Billstein, 2003) and another in 11th edition (Wheeler & Wheeler, 2005). This suggests a market that supports multiple textbooks over many years.[1]

In K-12 mathematics teaching, textbooks are an important influence on what is taught, and thus, what students have an opportunity to learn (Schmidt *et al.*, 2001;

Sosniak & Perlman, 1990; Stodolsky, 1989). The same may be true in these undergraduate courses: the textbooks may exert a major influence on the content and approach of courses for prospective elementary teachers. One important reason that textbooks may be influential in these classes is that they are often taught by inexperienced instructors.[2] The CBMS survey indicates that, in universities offering PhD's, graduate teaching assistants teach 31% of precalculus classes in universities, while tenure track faculty teach 17% of such classes. In addition, it is widely (albeit anecdotally) believed that most mathematics professors are not eager to teach classes for elementary teachers. Instructors who are new to a class or who are not committed to teaching the class may be more likely to depend on published materials. New questions on the 2005 CBMS survey will provide additional information about textbook use in such classes.

All of this is to argue that the content, format, and style of these textbooks may have a significant impact on what is taught and learned in mathematics courses for elementary teachers. In this article, we address two questions: 1) How do the contents of these books compare overall? 2) How do the books address three specific topics – introduction to fractions, multiplication, and reasoning and proof -- in what order, to what depth, and with what specific mathematical entailments?

Methods

We identified textbooks in print and, to the extent possible, in preparation through web searches, contacts with publishers, library searches and word of mouth. Some of the textbooks have extensive supplementary materials including such things as optional CD-ROMS, Web sites, practice books, and extended answer keys. We decided to include only the materials that are required for using the textbook, materials that would come with the textbook. For example, Masingila et al (2002) includes two volumes, as does Beckmann (2005).

Our analysis was conducted at two levels. First, we made an inventory of coverage in each book using tables of contents. We counted pages per chapter and laid out an overall comparison of contents in a table, indicating topics covered as chapters or sections of chapters; total pages; and average, minimum and maximum chapter lengths. We developed a map for each book showing what topics were covered and in what order. The expanded table (which includes books now out of print) and samples of the maps are available on the Web at http://www.msu.educ.edu/Meet/textanalysis.htm as Appendices A and B respectively.

Next, we identified three topics for in-depth analysis: fractions, multiplication, and reasoning and proof. The reasons for these choices are explained below. For these three topics, we developed analysis tables to record how each book handled the topic, each table unique to the topic. Analysis tables for fractions and multiplication include categories for definitions, sequence, coverage, representations, problems, and pedagogy. The reasoning and proof table is different from the others, for reasoning and proof may be integrated with other topics. Our analysis located occurrences of specific types of reasoning and proof such as proof by counterexample and logical rules of inference. The tables are also available at the url above as Appendix C.

Each book was analyzed and coded by at least two researchers, recording the coding in the tables. We discussed our codings, both to reach agreement and to maximize our understanding of the books. Using these tables we looked for similarities and differences across the books. Our method has been to propose hypotheses about the books and test against the data to see if our hypotheses hold. The categorizations of books in the tables below represent our collective opinion of how each book is situated given our definitions of the category.

Interesting to us is the fact that we began this study in 2004 with 21 textbooks in print, yet as of the end of 2005, there are only 14 such books, including one (Wheeler & Wheeler) that has been widely used for teacher education, but in earlier editions was aimed at a broader audience and had a different, more generic name. To the list of 14, we add the partial book by Wu that is not yet published. It is included in analyses where appropriate, given that it is an incomplete book. In the following sections, we discuss results of the two levels of analyses: overall content and detailed topics.

Overall Content

To understand the contents of the 14 published books, we used tables of contents and indexes to determine what topics are included. As shown in Table 1, there are many consistencies in coverage across the 14 books. Every book includes whole numbers, fractions and rational numbers, decimals, percents, operations, and number theory. All but one includes a chapter or sections on ratio and proportion. Except for number theory, this is all standard fare for K-8 mathematics, and thus not surprising to find in these books. Most of the books include the other topics as well: included in 10 of the 14 books are logic, number systems, and mental math. Included in 9 of the 14 is reasoning and proof.

***** Insert Table 1 about here *******

From this view, the books appear relatively consistent. The two books that are most different from the others – Parker and Baldridge (2004) and Jensen (2003) – do not include topics such as geometry and data because they are intended for a single semester course in number and operations, while other books can be used for two to four semesters. Parker and Baldridge have a second volume in preparation that will include many of the other topics in the table.

Although there is topical consistency, the length of the books, and the space devoted to different topics varies widely. For example, the average chapter length in these books varies from 25 pages (Parker and Baldridge) to 72 pages (Bennett and Nelson) with a mean of 52 pages across the 15 books. Similarly, the number of chapters ranges from 8 (Jones et al) to 17 (Musser et al) with a mean of 14. Because of differences in page and font size, and thus differences in the amount of content per page, it is not entirely accurate to compare textbook length.

***** Insert Table 2 about here *******

Table 2: Book data: length, chapters, editions

To further explore the global differences across these texts, we look at three dimensions of the books: coverage, presentation, and mathematical stance. *Coverage* assesses the comprehensiveness of the book, including what kind of content is presented. *Presentation* considers the organization and development of material. *Mathematical stance* considers how the books treat learning about mathematics and the treatment of meta-mathematical ideas. In the next sections, we explain and delineate these dimensions across the set of books.

Coverage

As is evident from Table 1, 11 of the 14 books have similar coverage of the subject of elementary mathematics at the level of chapter and section headings, and the three that differ are not written for a multiple semester sequence of courses. Across the books, even those with similar coverage overall differ at a more detailed level. Some books include details such as historical context, examples from K-8 curriculum, references to national (NCTM) standards, and illustration of pedagogical tools such as base-ten blocks or fraction bars. Other books focus only on mathematics with little motivational, pedagogical, or historical material. We call this dimension "coverage". It is not normative – that is, it is not inherently better or worse to be more or less extensive in coverage. Some argue that content such as historical development of a topic can lead to a better understanding of mathematics and/or more interest in the subject, while others say that extra details detract from the important focus on mathematics. We might say that the most *extensive* books are those with the greatest variety of information within topics, while the most *intensive* books are those that keep the focus squarely on the mathematics of the topic with little extra information. Arranging the books on this dimension yields groupings shown in Table 3.

******Insert Table 3 about here **********

- Extensive: includes historical references, references to NCTM standards, curriculum examples, and/or pedagogical tools throughout the book
- Mixed: includes some of the above, or all of them spread more sparsely in the book

- Intensive: focuses primarily on "pure" mathematics with little or no content of the sort listed above

Although these are qualitative rather than quantitative categories, they reveal important differences across the books, and raise some interesting questions about the publication process. Not only do the books in multiple editions tend to be longer, but they also tend to be more extensive. Looking at earlier editions of books, we see that authors may add content to be up-to-date (e.g., mentioning new standards documents or including references to communication or history of mathematics) without actually changing the mathematical substance of the book.[3]

Presentation

The books differ in how they organize and develop mathematical ideas. Presentation can range from *encyclopedic* to *narrative*. An encyclopedic book covers every topic with approximately the same degree of emphasis in sections of uniform length. The book may have an extensive index that can be used to look up any topic. In fact, such a book could be used as an encyclopedia of elementary mathematics, a reference book for a teacher.

By contrast, a narrative book presents a "story" of mathematics, giving different emphasis to topics depending on their importance to the curriculum of elementary mathematics or the overall understanding teachers need. In such a book, an index may not tell the whole story, since there are many connections across topics that are not easily tracked down one page at a time. For example, in one of the books that takes a problem solving approach – Masingila et al – the exposition of the mathematics is in a separate book from the problems. The expectation for using this textbook is that students will learn the mathematics from doing the problems, using the expository text as backup. The problem book itself is not indexed, although it has a table of contents identifying each problem with a mathematical topic. The problems themselves build a story of mathematics, made clear in the instructor's manual, while the expository text is a scaffold on which the story can be built.

A different narrative approach is found in Parker and Baldridge where the text is tied to a particular curriculum, the Singapore series for K-6 mathematics. The authors develop the mathematics through the lens of that curriculum, giving relative importance to concepts, topics, or approaches that are known to be difficult for elementary students and/or teachers. This book, and others on the narrative end of the scale, provide a coherent landscape of mathematics with hills and valleys representing topics or ideas that vary in both difficulty and importance.

While the category of "presentation" overlaps with "coverage" in some respects, it is not the same. For example, the Beckmann book is relatively extensive with respect to content, but is also a narrative book that defines a mathematical landscape. In some ways, this dimension measures what Cuoco has called "flatness" (Cuoco, 2001) – the books on the encyclopedic end of this scale give equal coverage to most topics making it hard to tell what is more or less important in the landscape of elementary mathematics.

This dimension is not normative – there are good reasons for choosing either type of textbook for use in a teacher education program. In particular, the narrative texts each reveal an articulated view of mathematics with which an instructor (or program) may not agree, limiting the usefulness of that text for that instructor. As an example, some mathematics instructors might argue that the Parker and Baldridge book is too dependent on the Singapore curriculum in a way that narrows the scope of the mathematics that is taught and learned. Or, they may have a mandate to include references to standards or curricula that are not available in specific narrative texts. On the other hand, the encyclopedic texts are more neutral with respect to the landscape of mathematics, and thus more amenable to different interpretations imposed by instructors, programs, or departments. With an encyclopedic book, the content, structure, and landscape of the course can be defined (narrated) by the instructors through their syllabi and daily lessons.

On the other hand, when using an encyclopedic textbook, an instructor may have difficulty developing a mathematical narrative for the course, or he may have difficulty deciding what to exclude. This may be particularly true for instructors who are inexperienced or uninterested in teaching preservice teachers.

Grouping the books on this dimension yields the following table. Note that there is considerable variation within each of these three categories. The boundaries between categories are not crisp.

*****Insert Table 4 about here *****

Encyclopedic: The book features comprehensive coverage with a complete index. Sections are similar in length. Treatment of topics is uniform in the text, and topic is the organizing principle of the book. The book is a reference for the mathematics of elementary school.

Mixed: More variation across topics is apparent. Coverage may be less complete, omitting or deemphasizing some topics. The book may be organized around concepts rather than topics. The index may be less comprehensive, or may not cover all occurrences of a word or topic. The book has more connections across topics and more of a sense of developing a mathematical terrain than the encyclopedic books.

Narrative: Sections are of different lengths based on the importance or difficulty of the ideas encountered. The book is an exposition of the mathematics of elementary school, developing a flow of ideas through logic or narrative. Connections between and among topics are emphasized, conveying a sense of the mathematical terrain.

Mathematical Stance

Textbooks start from different assumptions about the nature of mathematics and the knowledge prospective teachers bring to the course. These combine to create a distinct view of mathematics and mathematics learning that permeates each textbook. “Mathematical stance” addresses the conception of mathematics the book presents: What is important? What is the nature of mathematics? How does mathematics work as a discipline? This is perhaps the dimension of most variation across the books.

The mathematical stance of a textbook can be identified by the exposition of what might be called meta-mathematical ideas such as the role of definitions in mathematics, the nature of mathematical reasoning, the importance of precision in mathematical language, and the nature and use of assumptions in mathematical reasoning. The exposition of these ideas can be explicit, implicit, or absent; limited to a chapter (or chapters) on mathematical reasoning or some other topic, or pervasive throughout the book. We consider these differences in three main categories: Explicit, implicit and other, explained below.

For example, consider definitions. Given the significant role that definitions play in the teaching and learning of mathematics (e.g., Ball & Bass, 2000; Mariotti &

Fischbein, 1997; Vinner, 1991; Zaslavsky & Shir, 2005), we ask the following questions: Does the textbook use clearly stated definitions for fractions, multiplication, etc? Does the textbook use definitions in the exposition of topics? Does the textbook make explicit how definitions are used in the text and their role in doing mathematics?

In some books, definitions are consistently provided throughout the book, and their mathematical function is made *explicit*. That is, they are not only given and used, but the nature and role of definition in mathematics is discussed. In other books, definitions are provided but attention is not called to their function in mathematical reasoning. While the mathematical exposition may imply that definitions are important, their importance is *implicit*. In a third category are *other* textbooks in which definitions may not be given or used, or their use may be varied or inconsistent. All three of these approaches to definition are represented in this collection of textbooks. Differences such as these extend to other concepts as well.

Although there is room for argument about how and when mathematical definitions should be used, one could argue that the last option – no use or inconsistent use -- is mathematically questionable. As between explicit and implicit use of meta-mathematical ideas, there is no clearly correct approach. We might agree that knowing the importance and role of definition in mathematics is critical for those who teach mathematics, but it is not clear whether telling prospective teachers about this importance is more effective than giving them practice in using definitions in mathematical work.

Based on our work in the three topics presented below, we categorize the textbooks along this dimension of mathematical stance in Table 5. Even within these three categories, there are substantial differences across the books. For example, Jensen's book is a “theorem-proof” book giving a conventionally rigorous presentation of mathematics with brief explanations of meta-mathematical ideas. None of the other books take this approach. The Parker and Baldridge text, also classified here as “explicit,” builds up to definitions in stages, calling attention to the need for definitions appropriate to the students’ mathematical understanding. Their book is rigorous in a completely different way for they are careful to make the book mathematically correct, but not in a conventional form.

As shown in Table 5, most of the books fall into the “other” category, suggesting that underlying mathematical ideas and processes – mathematical ways of thinking – are neither explicit nor exemplified. Note that the categorizations in this table are based on our detailed analysis of three topics and thus may not hold true for the entire book.

*******Insert Table 5 about here *******

Content by Topic

Analysis at a global level reveals important distinctions among the books. Some differences, however, are more clearly seen at the level of topic. Examples include the relative importance of topics, the specific examples, definitions, and representations of a topic, how problems are situated and used, and the style and rhetoric of presentation. To explore some of these details, we turn to analysis across three important elements of the K-8 curriculum: introduction of fractions, multiplication, and reasoning and proof – a set of numbers, an operation, and a way of doing mathematics.

We choose these three topics because they represent different aspects of the K-8 curriculum and raise different issues for K-8 teaching and learning (National Council of Teachers of Mathematics, 2000). Fractions is a topic that recurs in K-8 mathematics, beginning in about grade 3 and continuing through grades 6-8 depending on the curriculum. Wu (1999; , 2005) argues that understanding fractions is critically important for the future study of algebra and more advanced mathematics and constitutes “students’ first serious excursion into abstraction” (2005, p. 2). Fractions have been a source of trouble for elementary teachers and students, a topic that seems to be poorly understood and much maligned (e.g., Behr *et al.*, 1983; Mack, 1990, 1995; Saxe *et al.*, 2005). Students may believe that operations on fractions are arbitrary and that the definition of a fraction changes depending on the context. They routinely fail to grasp the concept of a unit.

Multiplication is a different kind of topic in K-8 mathematics, an operation that is taught across grade levels from K-8 and appears in more complex versions as students’ exposure to number systems expands. Multiplication is taught first with whole numbers and gradually extended to fractions, decimals, percents, integers and sometimes, algebraic expressions in the course of the K-8 curriculum. Research suggests that students often come away from early mathematical experiences believing that multiplication changes in different number systems (Lampert, 1986). At the same time, they hold on to the misconception that “multiplication makes bigger” (e.g., Bell *et al.*, 1981).

Finally, reasoning and proof is currently not often an explicit topic in K-8 mathematics, taught implicitly if at all (Ball *et al.*, 2002). Yet, because reasoning and proof are critical elements of learning mathematics, there is a growing appreciation of the idea that reasoning should be part of all students’ mathematical experiences and across all grades (National Council of Teachers of Mathematics, 2000; Schoenfeld, 1994; Yackel &

Hanna, 2003). This places increased demands on K-8 teachers' understanding of reasoning and proof. In comparing the treatment of reasoning and proof in these texts, we investigate how textbooks for teachers approach reasoning and proof, whether implicit or explicit and in what detail.

In the next sections, we take up each of these topics as they are presented in the 15 textbooks. Although there is much to say, we limit the focus here to a single aspect of each topic. More detailed analysis is forthcoming in papers in preparation (Siedel & McCrory, in preparation; Stylianides & McCrory, in preparation).

Fractions

Fraction is a topic central to the K-8 curriculum. While understanding fractions is uniquely critical to a later understanding of algebra, we know from research on learning and on teacher knowledge that fractions is difficult for students and problematic for teachers (e.g., Ball, 1988; Behr et al., 1983; Ma, 1998; Mack, 1990, 1995; Saxe et al., 2005). Basic work with fractions begins as early as Kindergarten and continues throughout the elementary grades. The study of fractions is a major part of the elementary curriculum beginning in about third grade.

In our analysis, we consider how fractions are introduced in the textbooks. Introducing fractions to children and to prospective teachers means making connections between what they already know about fractions and the mathematics they need to know. This issue is especially critical for prospective teachers who may come to their mathematics classes with considerable misinformation about fractions, ideas that are incorrect or incomplete as well as beliefs that fractions are difficult or inscrutable. We start with the definition of fractions as given in these books.

Definitions are the backbone of mathematics. Some mathematicians have argued that one of the shortcomings of curricula for the elementary and middle grades is the absence of definitions and the failure to build mathematics from well-defined terms. Yet it is not obvious how to define mathematical objects at a level that is suitable for elementary students while retaining mathematical integrity. Even at the level of teacher education, this problem looms large, and fractions provides a compelling illustration of the complexity of defining terms mathematically while at the same time doing what makes sense pedagogically (McCrory, 2006). In the 15 textbooks, we see several approaches to this problem with respect to fractions.

For fractions in particular, the problem of definition is tied to what students (whether K-8 or undergraduate preservice teachers) bring to the classroom. Preservice teachers arrive with many years of schooling and plenty of exposure to fractions. They "know" what a fraction is. In their undergraduate mathematics classes they need to develop deeper knowledge about fractions, unravel misconceptions, and develop fluency in their use and understanding of fractions. Some books travel the road taken in the elementary curriculum, beginning with a part/whole definition using discrete objects. Others begin from an advanced perspective, developing the topic in a purely mathematical way. Still another group presents at once all the different ways of looking at fractions.

In Table 6, we show which definitions or models are included in the books. The models included are number line; part/whole; symbolic or ordered pair; ratio; and division. Examples are given after the table. In some books, fraction models are presented as three categories: set, length or linear measurement, and area or regional (Bassarear;

Masingila; O'Daffer; Parker and Baldridge) with "meanings" (or examples or uses) of fractions taken from a larger suite of examples or uses of fractions. For example, Bassarear discusses fraction as measure, quotient, operator and ratio using the three models (set, linear and area) to illustrate these four uses of fraction.

Take the example of fraction as division. Although 13 of the books talk about fraction as division (e.g., a/b = a÷b), only Jensen and Wu give explicit explanations of why this is true. A student or reader might be able to put an explanation together based on the contents of the book, yet the explanation is not explicit in other books. It is possible that such an explanation is somewhere in the book, not indexed and not included in the exposition of fraction. Such an explanation would answer the question "why does the fraction a/b have the same value as the division a÷b?" in terms of the models, representations, or definitions given. Most of the books seem to assume that this is true, or to assume that it is obvious without explanation.

Wu describes the problem this way: "[W]hen it is generally claimed that (for example) "the fraction 2/3 is also a division 2 ÷ 3", this sentence has no meaning because the meaning of 2÷3 is generally not given. A division of a number by another is supposed to yield a number, but, apart from the ambiguity of the meaning of a "number" in school mathematics, there is no explanation of what number would result from 2 divided by 3, much less why this number should be equal to a "part of a whole" which is 2/3." (Chapter 2, p. 32)

****Insert Table 6 about here ******

Examples:

A) **Number line** from Jensen 2003: The fraction p/q represents the point on the number line arrived at by dividing the unit interval into q equal parts and then going p of the parts to the right from 0. This point is called the *value* of the fraction. A *rational number* is the value of some fraction (p. 190).

B) **Part-Whole** from Darken 2003: Part-Whole definition of the elementary fraction: A/B refers to A parts of a quantity that is partitioned into B equal parts, where A and B are whole numbers, B ≠ 0 (p. 23).

C) **Ordered Pair** from Long and DeTemple 2006: A fraction is an ordered pair of integers a and b, b≠0, written $\frac{a}{b}$ or a/b (p. 343).

Symbolic from Billstein et al 2004: Numbers of the form a/b are solutions to equations of the form bx=a. This set, denoted Q, is the set of rational numbers and is defined as follows: Q={a/b | a and b are integers and b≠0}. Q is a subset of another set of numbers called fractions. Fractions are of the form a/b where b≠0 but a and b are not necessarily integers (p. 266).

D) **Ratio** from Bassarear 2005: A fraction is a number whose value can be expressed as the quotient or ratio of any two numbers a and b, represented as a/b, where b ≠ 0 (p. 266).

E) **Division** from O'Daffer 2005: A number is a **rational number** if and only if it can be represented by a pair of integers $\frac{a}{b}$, where b≠0 and a/b represents the quotient a÷b…. The symbol $\frac{a}{b}$ used in the previous definition is a fraction…. (p. 284).

In Table 7 we show whether each book makes a distinction between the symbol for a fraction and its value. In books that define fraction as a point on a number line, for example, this distinction becomes important since each (rational) point is represented by multiple fractions. As seen in the table, most books do not address this distinction.

Another category on Table 7 shows how each book relates fraction to rational number. Three books define fractions as a subset of rational numbers (sometimes using the term “elementary fractions”); five books define rational numbers as a subset of fractions; six books have the two sets equal; and one book does not relate the two explicitly. The issue is whether the term fraction extends to negative numbers, irrational numbers and/or expressions. Is (x+2)/x a fraction? Is -2/3 a fraction? These authors do not agree on answers to those questions.

Finally, we categorize the books as to their use of definitions and models. Five books use a definition to develop the concept of fraction and to connect the different models and representations. One book makes mathematical arguments to connect models, but does so developmentally starting from a simple conception and building up to a definition. Eight books present multiple models and representations without a primary definition and without specific mathematical explanations to connect them. One book uses a problem-based approach through which students may develop mathematical connections across representations.

****** Insert Table 7 about here *****

Multiplication

Multiplication permeates the K-8 curriculum, beginning early with whole number multiplication and continuing through fractions and integers and on into algebra. In our analyses, we consider overall treatments of multiplication of whole numbers, fractions, decimals, and integers and we look for ways that textbooks unify the concept of multiplication. In another paper, we focus on multiplication of integers as a particularly interesting place where mathematics and pedagogy often have a tug of war (Siedel & McCrory, in preparation).

As shown in Table 8, most of the books treat multiplication within number systems. Multiplication is introduced in a section on whole number operations and taken up sequentially in sections or chapters on integers, fractions and decimals later in the book. To make the connection across number systems, some of the books use an area model for multiplication, representing the product of two numbers m and n as the area of the rectangle enclosed by sides of length m and n. Although this representation extends from whole numbers to positive fractions and decimals, it becomes problematic for negative numbers. It also fails to address how other representations of multiplication, commonly introduced for whole numbers, are connected to multiplication of fractions and decimals.

Most books begin with a definition or representation of multiplication as repeated addition and include two other primary representations: multiplication as an area or array, and as a Cartesian product. Parker and Baldridge generalize these models as a set model, linear measurement model, and array or area model. This makes explicit the connection of representations of fractions and multiplication, suggesting a general scheme of representation through discrete, linear, and area models.

In the middle column of Table 8, we show the books that address explicitly the similarities and differences of multiplication across number systems. The books that make explicit connections go beyond using the same representation and showing that the same laws hold (commutative, associative, distributive). They attempt to explain why other representations are not used; they call attention to differences that might be problematic; or they try to extend the language of multiplication used for whole numbers to illustrate how it works in another number system. Beckmann, for example, focuses on use of the word "of" in defining multiplication: 1/3 • ½ means one-third **of** one-half just as 3•2 means three **of** two objects. Sometimes the words are awkward, but the point is to make a clear and explicit connection across number systems.

***** Insert Table 8 about here *****

Differences persist below the level of detail shown on this table. For example, the Sonnabend text includes area and Cartesian product models of multiplication, but limits coverage to half a page during the initial exposition of multiplication of whole numbers. They are referenced only briefly elsewhere in the book and not used in connection with multiplication of integers, fractions, or decimals. By contrast, Beckmann explains models for multiplication across fifteen pages, with continuing use of the models over an additional twenty pages.

Another example of difference in detail is in the use of names for models. Several books identify set, measurement, and area models for multiplication. Musser, on the other hand, illustrates set and measurement models and then extends them to array and area, naming them respectively as set and measurement models in two dimensions.

Reasoning and Proof

Our third topic, reasoning and proof, is different from the others because it has traditionally had a limited role in the K-8 mathematics curriculum, taught implicitly if at all (e.g., Ball *et al.*, 2002). However, because reasoning and proof are critical elements of learning mathematics, there has been a growing appreciation of the idea that reasoning and proof should be a central part of the entire school mathematics curriculum (Ball & Bass, 2003; Hanna, 2000; National Council of Teachers of Mathematics, 2000; Schoenfeld, 1994; Yackel & Hanna, 2003). This places increased demands on K-8 teachers' knowledge of reasoning and proof but also on mathematics courses for preservice K-8 teachers because existing research shows that these teachers face significant difficulties in understanding logical principles and distinguishing between empirical and deductive forms of argument (cf., Chazan, 1993; Knuth, 2002; Martin & Harel, 1989; Morris, 2002; Simon & Blume, 1996; Stylianides *et al.*, 2004). It seems unlikely that prospective K-8 teachers will develop adequate knowledge of reasoning and proof unless the mathematics courses they take offer them the opportunities to develop this knowledge. Examination of the treatment of reasoning and proof in the textbooks of our study will suggest the extent to which these prospective teachers might learn about reasoning and proof in courses that use these texts. Thus, we examined whether textbooks for teachers made reasoning and proof an explicit topic, and if not, where and how reasoning and proof appear in these texts.

Unlike the other focal topics in our research, however, reasoning and proof is not necessarily a separate topic but may be a theme or strand throughout a text; or, it may be something that is mentioned only sporadically. To analyze different approaches to presenting reasoning and proof, we first identified, from a mathematical standpoint, major elements or components of this topic and created a list of concepts and topics related to reasoning and proof, and then we looked for them in textbooks in two ways. First, we looked in the table of contents to identify specific chapters or sections where these topics were included. Second, we used book indexes to find pages where these topics were mentioned. Through these two approaches, we identified as many occurrences of proof-related topics as we could find.

Several different approaches emerged, with reasoning and proof found in the following ways:

1. In a chapter with reasoning or proof in the title
2. In a chapter or section on logic
3. In a chapter on problem solving
4. In some other chapter or chapters, or throughout the book
5. Not explicitly covered

**** Insert Table 9 about here *****

In Table 10, we indicate the number of references to words associated with reasoning and proof indexes of the books. Two books have no index – Wu (in preparation) and Jones (2000). Most often, the references in the index pointed to the chapters in which reasoning and proof were taught. That is, there are few references to any of these terms outside of the sections specifically aimed at teaching reasoning and proof. The terms "mathematical induction" and "explanation" were included in none of the indices and are not shown on the table. The finding with regard to "explanation" is particularly interesting because it is an explicit focus in many of the textbooks. For example, Beckmann (2005) devotes half of her first chapter to a section on "Explaining Solutions," discussing explicitly what constitutes a good explanation in mathematics and what it means to write a good explanation. Yet, she does not include "explanation" in the index, even though the index is extensive and includes also several non-mathematical terms. Only two of the books – Parker and Baldridge (2004) and Bennett and Nelson

(2004) – have the word “definition” in the index, and only one – Darken (2003) – has “assumption” in the index. These omissions are telling, suggesting that a student using the textbook might have a hard time learning about definitions or assumptions except by experiencing their use in the class.

Tables of contents and indices aside, the books differ noticeably in how they approach reasoning and proof. On the one hand, Jensen (2003) is a definition-theorem-proof book in the style of a classic mathematics text. In other books, there are almost no proofs to be found (although nearly every book proves the Pythagorean Theorem), and even mathematical reasoning is hard to point to. In between these are books such as Parker and Baldridge, Beckmann, and Darken that make careful mathematical arguments and sometimes offer proofs, but not to the level of formality found in Jensen. These differences raise a question about what it means for a textbook to be rigorous. Can we define a kind of rigor that applies to books that eschew formal mathematics, or is rigor only found in a book like Jensen’s? If we do not call it rigor unless it is formal, what term can we use to distinguish among books that do and do not present mathematics through careful mathematical argumentation and reasoning?

****** Insert Table 10 about here ******

Discussion and Conclusions

All of these textbooks are written to be read and used, yet one of the challenges and dilemmas of teaching mathematics at all levels is that students do not know how to read and learn from a mathematics textbook. It is common to hear from students that they use the book only to get the homework problems, sometimes looking back at a specific section to see how to work a problem. Without exception, these books are addressed to an audience of readers, meant to be read carefully. It is clear that words, illustrations, and examples are carefully chosen. Perhaps part of the job of teaching courses to prospective teachers is helping them learn to learn mathematics from a book. It will almost certainly be part of their job as teachers if they take seriously the work of making mathematics make sense for their students.

At the same time, the books that are encyclopedic and extensive are very hard to read, even for the group of researchers who analyzed the books for this study. The mathematics is mixed in with other things – history, curriculum, standards, puzzles, and more – making it hard for a prospective instructor of the course to navigate the book. What do you focus on? How do you use the sidebars and excursions away from the mathematics? We found that even some features that should be advantages were distractions: as used in some of the books, multiple colors, for example, made it harder to follow the flow of mathematics. There has been some work on learning from texts suggesting that the features and characteristics of text that support learning are complexly interrelated (Kintsch, 1986). In mathematics, this is an area that is ripe for research.

In spite of these problems – that the students for whom the texts are intended may not use the text as a site for learning, and that the design of mathematics textbooks can impede learning – a basic assumption of our research is that textbooks define in large part what students have an opportunity to learn. For example, students are not likely to learn that fraction can be defined or modeled via sets, linear measurement, or area if those distinctions are not presented in the book. Although an instructor can add to what is offered in the textbook, or can skip specific ideas, sections, or chapters, it is most likely that the textbook, when used for a course, defines the boundaries of what students are offered. In other words, the textbook is used to create the intended curriculum for a course that uses it.

An unknown that emerges from our study is the impact of explicit versus implicit exposition of particular aspects of mathematics. An example is mathematical reasoning. How does a student best learn about mathematical reasoning – e.g., what constitutes a mathematical argument, what is the appropriate level of detail, what is the role of assumptions and definition, etc? In most of the books, the assumption seems to be that students learn these things by example, by seeing them done, but without explicit attention to how they are constituted. It is not clear from research whether this is the best way to teach about mathematics.

Another important example where implicitness is common in these books is in making connections across topics. Many of the books are written as if clear explanation of each topic individually is enough for students to make connections across topics. Multiplication is a good example. In multiplication of whole numbers, explanations are provided for different interpretations or models. Yet, in several books, by the time decimals are introduced, the meaning of multiplication is no longer attended to and the relationship of decimal multiplication to the early models is not mentioned. In later sections, multiplication is presented as a procedure to be followed. It is a pattern in many books to attend to meaning in the earliest (and possibly easiest) sections on any given topic, and to move to algorithmic details as the book proceeds and complexity grows. In some cases, the sidebars on history, standards, and the like may create the appearance that the book is attending to understanding, while the actual mathematics on the page is in the form of procedure after procedure.

Yet another example where implicitness is common is in making connections between learning mathematics and learning mathematics *for teaching*. Reasoning and proof is a good example. Several textbooks cover concepts related to reasoning and proof with little motivation or connection to the work that teachers do in classrooms. For example, why does a preservice elementary *teacher* need to understand proof by contradiction or proof by counterexample? By teaching these proof methods on the grounds only of their mathematical/logical significance, preservice teachers are not supported, in an explicit way, to appreciate also their teaching significance. We hypothesize that explicit links to the work of teaching through the study of samples of student work or segments of classroom instruction where issues of refuting mathematical claims are central (Carpenter *et al.*, 2003; Lampert, 1990, 1992; Reid, 2002; Zack, 1997) can help preservice teachers appreciate the importance of knowing proof by contradiction and proof by counterexample, making it also more likely that they will use this knowledge in their teaching.

What have we learned from our analysis? Our most important lessons are these:

1. The differences across books in how topics are presented are important and can be mathematically conflicting. An example is the relationship between rational numbers and fractions. A prospective teacher may encounter contradictory information, for example, if she learns in her math class that rational numbers are a subset of fractions and then her curriculum says the opposite (or vice versa). At a minimum, books should equip teachers to make sense of such conflicts, perhaps by addressing them directly.

2. Although mathematics is a subject in which right and wrong can be assessed, such assessment is always based on a set of accepted facts (such as definitions) and

canons of correct inference. When assumptions differ, as they can and do, ambiguity can result. The books that neglect the ambiguous (e.g., that "fraction" does not have a universally agreed upon meaning) are doing a disservice to future teachers. Some books go too far in the direction of presenting mathematics as a closed, cut-and-dried, right and wrong subject even in the face of ambiguity such as the word fraction presents, while others go too far toward presenting mathematics as a collection of possibilities in which there is no ultimate authority. An example is the use of models for integer multiplication, a subject that is analyzed in more detail in a forthcoming paper (Siedel and McCrory, in preparation). In this case, books that use models to represent multiplication of a negative times a negative (e.g., the chip model) neglect to explain or present the failures of the models, both mathematically and as a realistic application of negative numbers.

3. The books take very different positions with respect to the mathematics of elementary curricula. For example, should these books, and the courses in which they are used, be the place where students learn about the mathematical affordances and limitations of models like the chip model for multiplying integers? Or should that kind of work be done in a methods class?

4. The assumption of many books is that representations speak for themselves, and in particular, that connections between representations of the same idea need not be made explicit. Models of fractions are a good example of this. In some books, a part-whole picture, a number line, and a rectangular area are all shown within a page or two, with no attempt to explain how they represent the same thing mathematically. The assumption may be that the future teachers already know what a fraction is, and what they need to learn is how to show different representations of it. An alternative

assumption would be that these students do not have a firm grasp of what a fraction is and the representations can be used as a tool to help them understand the meaning of fraction.

As we continue looking at textbooks for teachers, we continue to learn about the mathematics and about the complexity of designing mathematics curricula for future teachers. One of the ongoing issues for textbook authors and those who teach courses to prospective elementary teachers is trying to see the mathematics through the eyes of the student. What do they know about fraction or integer multiplication? What are their conceptions of multiplication as an operation? How do they think about mathematical reasoning or proof? Of course, they do not all think alike, but putting ourselves in their shoes is an interesting exercise as we try to understand how they might make meaning of the mathematics in these textbooks.

List of Textbooks

Bassarear, T. (2005). *Mathematics for elementary school teachers, 3rd edition* (3rd ed.). Houghton Mifflin.

Beckmann, S. (2005). *Mathematics for elementary school teachers* (1st ed.). Boston, MA: Pearson/Addison Wesley.

Bennett, A., & Nelson, L. T. (2004). *Math for elementary teachers: A conceptual approach*, (6th ed.). McGraw-Hill.

Billstein, R., Libeskind, S., & Lott, J. W. (2004). *A problem solving approach to mathematics for elementary school teachers* (8th ed.). Boston, MA: Addison Wesley.

Darken, B. (2003). *Fundamental mathematics for elementary and middle school teachers*: Kendall/Hunt.

Jensen, G. R. (2003). *Arithmetic for teachers: With applications and topics from geometry*: American Mathematical Society.

Jones, P., Lopez, K. D., & Price, L. E. (2000). *A mathematical foundation for elementary teachers* (Revised ed). New York: Addison Wesley Higher Education.

Long, C. T., & DeTemple, D. W. (2006). *Mathematical reasoning for elementary teachers, 4th edition* (4th ed.): Addison Wesley.

Masingila, J. O., Lester, F. K., & Raymond, A. M. (2002). *Mathematics for elementary teachers via problem solving*: Prentice Hall.

Musser, G. L., Burger, W. F., & Peterson, B. E. (2003). *Mathematics for elementary school teachers: A contemporary approach* (6th ed.). New York: John Wiley & Sons.

O'Daffer, P., Charles, R., Cooney, T., Dossey, J., & Schielack, J. (2005). *Mathematics for* *elementary school teachers, 3rd edition*. Boston: Pearson Education.

Parker, T. H., & Baldridge, S. J. (2004). *Elementary mathematics for teachers (volume 1)*. Okemos, MI: Sefton-Ash Publishing.

Sonnabend, T. (2004). *Mathematics for elementary teachers: An interactive approach for grades k-8* (3rd ed.). Brooks/Cole.

Wheeler, R. E., & Wheeler, E. R. (2005). *Modern mathematics for Elementary Educators* (11th ed.), Kendall/Hunt Publishing Company.

Wu, H. H. (in preparation). Chapter 1: Whole Numbers; Chapter 2: Fractions

References

Ball, D. L. (1988). *Knowledge and reasoning in mathematical pedagogy: Examining what prospective teachers bring to teacher education.* Unpublished dissertation, Michigan State University, East Lansing.

Ball, D. L., & Bass, H. (2000). Making believe: The collective construction of public mathematical knowledge in the elementary classroom. In D. C. Phillips (Ed.), *Constructivism in education*. Chicaqgo: University of Chicago Press.

Ball, D. L., & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 27-44). Reston, VA: National Council of Teachers of Mathematics.

Ball, D. L., Hoyles, C., Jahnke, H. N., & Movshovitz-Hadar, N. (2002). The teaching of proof. In L. I. Tatsien (Ed.), *Proceedings of the international congress of mathematicians* (Vol. III). Beijing: Higher Education Press.

Behr, M. J., Lesh, R. A., Post, T. R., & Silver, E. A. (1983). Rational-number concepts. In R. A. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 91-126). New York: Academic Press.

Bell, A., Swan, W., & Taylor, G. (1981). Choice of operation in verbal problems with decimal numbers. *Educational Studies in Mathematics, 12*(399-420).

Brumfiel, C. F., & Krause, E. F. (1968). *Elementary mathematics for teachers*. Reading, Mass.,: Addison-Wesley Pub. Co.

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic & algebra in elementry school*. Portsmouth, NH: Heinemann.

Chazan, D. (1993). High school geometry students' justification for their views of empirical evidence and mathematical proof. *Educational Studies in Mathematics, 24*, 359-387.

Conference Board of the Mathematical Sciences. (2001). *The mathematical education of teachers* (Vol. 11). Providence, RI: American Mathematical Society.

Cuoco, A. (2001). Mathematics for teaching. *Notices of the AMS, 48*(2), 168-174.

Freeman, D., Kuhs, T., Porter, A., Floden, R., Schmidt, W., & Schwille, J. (1983). Do textbooks and tests define a national curriculum in elementary school mathematics? *Elementary School Journal, 83*, 501-514.

Hanna, G. (2000). Proof, explanation and exploration: An overview. *Educational Studies in Mathematics, 44*, 5-23.

Kintsch, W. (1986). Learning from text. *Cognition and instruction, 3*(2), 87-108.

Knuth, E. J. (2002). Secondary school mathematics teachers' conceptions of proof. *Journal for Research in Mathematics Education, 33*(5), 379-405.

Krause, E. F. (1991). *Mathematics for elementary teachers: A balanced approach*. Lexington, MA: D.C. Heath and Company.

Lampert, M. (1986). Knowing, doing, and teaching multiplication. *Cognition and Instruction, 3*(4), 305-342.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*(1), 29-63.

Lampert, M. (1992). Practices and problems in teaching authentic mathematics. In F. K. Oser, A. Dick & J.-L. Patry (Eds.), *Effective and responsible teaching: The new synthesis* (pp. 458). San Francisco: Jossey-Bass publishers.

Lutzer, D. J., Maxwell, J. W., & Rodi, S. B. (2002). *Statistical abstract of undergraduate programs in the mathematical sciences in the united states: Fall 2000 cbms survey*: American Mathematical Society.

Ma, L. (1998). *Knowing and teaching elementary mathematics: teachers' understanding of fundamental mathematics in china and the united states*. Mahwah, NJ: Lawrence Erlbaum.

Mack, N. K. (1990). Learning fractions with understanding: Building on informatl knowledge. *Journal for Research in Mathematics Education, 21*(1), 16-32.

Mack, N. K. (1995). Confounding whole-number and fraction concepts when building on informal knowledge. *Journal for Research in Mathematics Education, 26*(5), 422-441.

Mariotti, M. A., & Fischbein, E. (1997). Defining in classroom activities. *Educational Studies in Mathematics, 34*, 219-248.

Martin, W. G., & Harel, G. (1989). Proof frames of preservice elementary teachers. *Journal for Research in Mathematics Education, 20*, 41-51.

McCrory, R. (2006). Mathematicians and mathematics textbooks for prospective elementary teachers. *Notices of the AMS, 53*(1), 20-29.

Morris, A. K. (2002). Mathematical reasoning: Adults' ability to make inductive-deductive distinction. *Cognition and Instruction, 20*(1), 79-118.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Porter, A., Floden, R., Freeman, D., Schmidt, W., & Schwille, J. (1988). Content determinants in elementary mathematics. In D. Grouws & T.Cooney (Eds.), *Perspectives on research on effective mathematics teaching,* (pp. 96-113). Hillsdale, N.J.: Lawrence Erlbaum Publishers.

Reid, D. (2002). Conjectures and refutations in grade 5 mathematics. *Journal for Research in Mathematics Education, 33*(1), 5-29.

Saxe, G. B., Taylor, E. V., MacIntosh, C., & Gearhart, M. (2005). Representing fractions with standard notation: A developmental analysis.

Schmidt, W. H., McKnight, C. C., Houang, R. T., Wang, H. C., Wiley, D. E., Cogan, L. S., et al. (2001). *Why schools matter: Using timss to investigate curriculum and learning*. San Francisco, CA: Jossey-Bass Publishers.

Schmidt, W. H., McKnight, C. C., & Raizen, S. A. (1997). *A splintered vision: An investigation of u. S. Science and mathematics education*. Dordrecth: Kluwer Academic Publishers.

Schoenfeld, A. H. (1994). What do we know about mathematics curricula? *Journal of Mathematical Behavior, 13*, 55-80.

Siedel, H., & McCrory, R. (in preparation). Multiplication of integers in textbooks for teachers: What good are models?

Simon, M. A., & Blume, G. W. (1996). Justification in the mathematics classroom: A study of prospective elementary teachers. *Journal of Mathematical Behavior, 15*, 3-31.

Sosniak, L. A., & Perlman, C. L. (1990). Secondary education by the book. *Journal of Curriculum Studies, 22*(5), 427-442.

Stake, R. E., & Easley, J. A. (1978). *Case studies in science education*. Urbana, IL: Center for Instructional Research and Curriculum Evaluation University of Illinois at Urbana-Champaign.

Stodolsky, S. S. (1988). *The subject matters: Classroom activity in math and social studies*. Chicago: University of Chicago Press.

Stodolsky, S. S. (1989). Is teaching really by the book? In P. W. Jackson & S. Haroutinunian-Gordon (Eds.), *From socrates to software: The teacher as text and the text as teacher* (Vol. 89, pp. 159-184). Chicago, IL: National Society for the Study of Education.

Stylianides, A. J., & McCrory, R. (in preparation). Knowledge of refutation for teaching k-8 mathematics: An analysis of mathematics textbooks for teachers.

Stylianides, A. J., Stylianides, G. J., & Philippou, G. N. (2004). Undergraduate students' understanding of the contraposition equivalence rule in symbolic and verbal contexts. *Educational Studies in Mathematics, 55*(133-162).

Vinner, S. (1991). The role of definitions in the teaching and learning of mathematics. In D. Tall (Ed.), *Advanced mathematical thinking* Dordrecht, Netherlands: Kluwer Academic Publishers.

Wu, H.-H. (1999). Some remarks on the teaching of fractions in elementary school. from http://math.berkeley.edu/~wu/fractions2.pdf

Wu, H.-H. (2005, April). *Key mathematical ideas in grades 5-8.* Paper presented at the National Council of Teachers of Mathematics Research Presession. Anaheim, CA:

Yackel, E., & Hanna, G. (2003). Reasoning and proof. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 227-236). Reston, VA: National Council of Teachers of Mathematics.

Zack, V. (1997). *"You have to prove us wrong:" Proof at the elementary school level.* Paper presented at the 21st Conference of the International Group for the Psychology of Mathematics Education, 291-298. Lahti, Finland: University of Helsinki

Zaslavsky, O., & Shir, K. (2005). Students' conceptions of a mathematical definition. *Journal for Research in Mathematics Education, 36*(4), 317-346.

Table 1: *Textbook Chapter and Section Topics from the Table of Contents*

	Bassarear (2005)	Beckmann (2005)	Bennett & Nelson (2004)	Billstein et al. (2004)	Darken (2003)	Jensen (2003)	Jones et al. (2000)	Long & DeTemple (2003)	Masingila et al. (2002)	Musser et al. (2003)	O'Daffer et al. (2002)	Parker & Baldridge (2004)	Sonnabend (2004)	Wheeler & Wheeler (2005)
Problem Solving	(1)	1	1	1	(2)			1	1	1	(1)	2	(1)	1
Sets	(2)		2	2	(1)	(1)	(8)	2	(1)	2	2		2	3
Reasoning and Proof	(1)		2	(5)	2(1,5)				(1)		(1,7)	5	1	(1)
Logic				(1)	(2)				(1)					2
Operations (+,-.x.÷)*	3(5)	4 +,- 5,6,7 x,÷	(3,5,6)	(2,3 4,5)	4(2) +,- 5,6(2) x,÷	(2,3,5, 6,7,8)	2	(2,3,5, 6,7)	3(7)	3,4 (6,7,8)	(2, 3,5,6)	1,3(6,8)	(3,5,6, 7)	(4, 6, 7)
Number Systems or Numeration	5	2(3)	(3)	3	1(3)	(3)	8	3(6)	2	2	(2,6)		(3)	4
Whole Numbers		(2,5)	3	2,3	(1,6)	2,3(1)	(1,2,8)	2(3)	3	2,3,4	2	1	3	4
Fractions	(5)	3(4,6,7)	5	5	(1,4)	(5)	(1,2)	6	6 (7)	6	(6)	6	6	
Integers	(5)	(2)	5	4	(1,6)	7	(8)	5	3	8	5	8	5	5
Decimals	(5)	(2,5,6,7)	6	6	(4,6)	6	(1,2)	7	(7)	7	(6)	9	7	(7)
Rational numbers	(5)	(2,12)	6	5	(1,3,6)	5 (7)	1(8)	6	7	9	6	9	6	6
Real numbers	(5)	(2)	(6)	6	2 (6)			7	(7)	9		9	7	7
Percents	(6)	(3,4)	(6)	6		(6)	(1)	(7)	(7)	7	(7)	7	7	(7,8)
Number Theory	4	12	4	4	7	4	(1)	4	4	5	4	5	4	5
Data or Statistics	7	14	7	8	8		6	9	5	10	8		12	10

Table 1: *Textbook Chapter and Section Topics from the Table of Contents*

	Bassarear (2005)	Beckmann (2005)	Bennett & Nelson (2004)	Billstein et al. (2004)	Darken (2003)	Jensen (2003)	Jones et al. (2000)	Long & DeTemple (2003)	Masingila et al. (2002)	Musser et al. (2003)	O'Daffer et al. (2002)	Parker & Baldridge (2004)	Sonnabend (2004)	Wheeler & Wheeler (2005)
Probability	(7)	15	8	7	9		7	10	5	11	9		13	9
Geometry	8,9	8,11	9	9,10	10		4,5	11, 14	9	12,14, 15	10		8	11
Measurement	10	10	10	11	11 (1,3)	(1)	3	12	10	13	12		10	12
Transformation (geometry)	9	9	11	12	12			13	(10)	16	11		9	13
Functions	(2)	13	2	2				(8)	8	(2,9)			2	(14)
Algebraic thinking or early algebra	(2)	13(4,5)	(1)	(1)				8		9	13	4(6,8)	11	
Mental math	(3,5)	(4,5)		(3)	(4)		(1)	(3)		4	(3)	2	(3,7)	
Estimation	(3,5,6)			(3)	(4)			(3)		(4)	3	(3)		
Ratio and Proportion	6	(7)	(6)	(5)	(1,5)	(5	(1)	(7)	(7)	7	7	(7)	(7)	

# indicates a chapter with the topic in the chapter name

(#) indicates a section within a chapter with the topic in the section name

* Two books – Beckmann and Darken – have separate chapters in which they treat +,- and x,÷ respectively. IN the other books, all four operations appear as subtopics together in several chapters.

NOTE: Blanks do not imply that the topic is not covered in the book, only that it is not specifically included in a chapter or section heading. The basis for the table is the most detailed version of the Table of Contents in each textbook.

Table 2: *Overall Length, Chapter Lengths, and Edition*

	Total pages	Max	Min	Avg	# Chapters	Edition
Bassarear (2005)	704	98	38	70	10	3
Beckmann (2005)	700	75	12	47	15	1
Bennett & Nelson (2004)	797	95	40	72	11	6
Billstein et al. (2004)	790	87	53	66	12	8
Darken (2003)	736	98	32	61	12	1
Jensen (2003)*	383	68	14	42	9	1
Jones (2000)	316	65	30	38	8	1
Long & DeTemple (2006)	946	87	53	68	14	3
Masingila et al. (2002)	492	71	11	49	10	1
Musser et al. (2003)	1116	73	31	59	16	6
O'Daffer et al. (2002)	931	82	42	64	13	3
Parker & Baldridge (2004)*	237	37	16	26	9	1
Sonnabend (2004)	787	95	13	61	13	2
Wheeler and Wheeler (2002)	712	66	32	51	14	11
Wu (not included in calculations)	227				2	
AVERAGE	689	65	25	45	9.8	2.8

Note: The book lengths include only the primary text, not supplemental material such as teacher's editions, problems booklets, or CD-ROM material.

* These two books are for a one semester course covering primarily number and operations; the others are for two or more semesters.

Table 3: *Textbook Coverage*

	Extensive	Mixed	Intensive
Bassarear, 3rd Edition (2005)		✓	
Beckmann 1st Edition (2005)		✓	
Bennett & Nelson 6th Edition (2004)	✓		
Billstein et al. 9th Edition (2004)	✓		
Darken 1st Edition (2003)		✓	
Jensen 1st Edition (2003)			✓
Jones et al. 1st Edition (2000)			✓
Long & DeTemple 3rd Edition (2003)	✓		
Masingila et al. 1st Edition (2002)		✓	
Musser et al. 6th Edition (2003)	✓		
O'Daffer et al. 3rd Edition (2002)	✓		
Parker & Baldridge 1st Edition (2004)			✓
Sonnabend 2nd Edition (2004)		✓	
Wheeler & Wheeler 11th Edition (2005)	✓		
Wu (2002)			✓

Table 4: *Presentation in Textbooks*

	Encyclopedic	Mixed	Narrative
Bassarear (2005)		✓	
Beckmann (2005)		✓	
Bennett & Nelson (2004)	✓		
Billstein et al. (2004)	✓		
Darken (2003)			✓
Jensen (2003)			✓
Jones et al. (2000)			✓
Long & DeTemple (2003)	✓		
Masingila et al. (2002)			✓
Musser et al. (2003)	✓		
O'Daffer et al. (2002)	✓		
Parker & Baldridge (2004)			✓
Sonnabend (2004)	✓		
Wheeler & Wheeler (2005)	✓		
Wu (2002)			✓

Table 5: *Mathematical Stance -- Attention to "Metamathematical" Ideas*

	Explicit	Implicit	Other
Bassarear (2005)		✓	
Beckmann (2005)		✓	
Bennett & Nelson (2004)			✓
Billstein et al. (2004)			✓
Darken (2003)	✓		
Jensen (2003)	✓		
Jones et al. (2000)			✓
Long & DeTemple (2003)			✓
Masingila et al. (2002)		✓	
Musser et al. (2003)			✓
O'Daffer et al. (2002)			✓
Parker & Baldridge (2004)	✓		
Sonnabend (2004)			✓
Wheeler & Wheeler (2005)			✓
Wu (2002)	✓		

Table 6: *Models and Definitions of Fractions*

	(A) Number Line	(B) Part/Whole	(C) Symbolic or Ordered Pair	(D) Ratio	(E) Division
Bassarear (2005)	✓	✓		P	✓
Beckmann (2005)	✓	P			✓
Bennett & Nelson (2004)	✓	✓		✓	✓
Billstein et al. (2004)	✓	✓	P	✓	✓
Darken (2003)	✓	P		✓	✓
Jensen (2003)	P	✓		✓	✓
Jones et al. (2000)		P		✓	✓
Long & DeTemple (2003)	✓	✓	P		
Masingila et al. (2002)		✓		✓	✓
Musser et al. (2003)		✓	P		P
O'Daffer et al. (2002)	✓	✓	P	✓	P
Parker & Baldridge (2004)	✓	✓			✓
Sonnabend (2004)	✓	✓	P		✓
Wheeler & Wheeler (2005)	✓	✓	P	✓	
Wu (2002)	P	✓		✓	✓

Notes:

P = Primary definition, √ = representation or model used in the text

I = Implied distinction between a the symbol and the value it represents, not made explicit.

Table 7: *Other characteristics of the introduction of fractions*

	Symbol v. Value	Rational Number v. Fraction	Category
Bassarear (2005)	✓	b	1
Beckmann (2005)	Implicit	b	1
Bennett & Nelson (2004)	✓	b	3
Billstein et al. (2004)		b	3
Darken (2003)		a	1
Jensen (2003)	✓	d	1
Jones et al. (2000)		c	3
Long & DeTemple (2003)		d	3
Masingila et al. (2002)		d	4
Musser et al. (2003)	✓	a	3
O'Daffer et al. (2002)	✓	d	3
Parker & Baldridge (2004)		d	2
Sonnabend (2004)		b	3
Wheeler & Wheeler (2005)		a	3
Wu (2002)		d	1

a. Distinguishes fractions (in some books, "elementary fractions") as a subset of the rational numbers.

b. Distinguishes rational numbers as a subset of fractions.

c. Section on rational numbers does not mention fractions.

d. "Fractions" refers only to rational numbers.

Categories:

1. Primary definition is used to develop the concept of fraction and is connected by mathematical arguments to other representations or models
2. Definitions, representations and models are explicitly connected by mathematical arguments, but there is not a primary definition used to make these connections.
3. Definitions, representations and models are connected intuitively without specific mathematical arguments.
4. Connections are made through problems in the problem book.

Table 8: *Multiplication in the Textbooks*

	Multiplication is found:			Definition(s) and models of multiplication		
	Within Number Systems	Across Number Systems (e.g., single chapter for all multiplication)	Explicit attention to similarities and differences across number systems	Repeated addition	Area or array	Cartesian product
Bassarear (2005)		✓	✓	✓	✓	✓
Beckmann (2005)		✓	✓	P	✓	✓
Bennett & Nelson (2004)	✓		✓	P	✓	✓
Billstein et al. (2004)	✓			✓	✓	✓
Darken (2003)		✓	✓	P	✓	✓
Jensen (2003)		✓		P	✓	✓
Jones et al. (2000)		✓		✓	P	
Long & DeTemple (2003)	✓			P	✓	✓
Masingila et al. (2002)	✓			✓	✓	✓
Musser et al. (2003)	✓			✓	✓	✓
O'Daffer et al. (2002)	✓			✓	✓	✓
Parker & Baldridge (2004)	✓		✓	✓	✓	
Sonnabend (2004)	✓			P	✓	✓
Wheeler & Wheeler (2005)	✓			✓	✓	✓
Wu (2002)						

Table 9: *Reasoning and Proof in the Chapters*

	Chapter on Reasoning and Proof	Chapter on Logic	Chapter on Problem Solving	Other	Not explicitly covered
	1	2	3	4	5
Bassarear (2005)				✓[a]	
Beckmann (2005)			✓		
Bennett & Nelson (2004)	✓		✓		
Billstein et al. (2004)		✓	✓		
Darken (2003)	✓			✓[b]	
Jensen (2003)				✓[c]	
Jones et al. (2000)					✓[g]
Long & DeTemple (2003)			✓		
Masingila et al. (2002)			✓		
Musser et al. (2003)	✓	✓			
O'Daffer et al. (2002)				✓[d]	
Parker & Baldridge (2004)				✓[e]	
Sonnabend (2004)	✓				
Wheeler & Wheeler (2005)		✓	✓		
Wu (2002)				✓[f]	

[a.] Bassarear has a chapter called "Foundations for Learning Mathematics" that includes reasoning and proof.

[b.] Darken treats reasoning and proof as a strand throughout the book, including a section at the end of each chapter calling attention to NCTM standards addressed.

[c.] Jensen discusses proof in the preface and uses proofs extensively throughout the book.

[d.] O'Daffer includes reasoning and proof in a chapter called "Mathematical Processes."

[e.] Parker and Baldridge include reasoning and proof in a chapter on number theory.

[f.] Wu's book pays explicit attention to reasoning and proof in the two completed chapters, both constructing proofs and discussing them.

[g.] Jones et al do not have a separate chapter that covers reasoning and proof, and the book has no index. There is a reference to proof in a chapter on geometry.

Table 10: *Reasoning and Proof in the Index*

	Index	Glossary	Assumption	Compound Statements	Conditional Statements	Proof by Contradiction	Contrapositive	Converse	Counterexample	Deductive Reasoning	Definition	Law of Detachment	Indirect Proof	Indirect Reasoning	Inductive Reasoning	Proof	Reasoning
Bassarear (2005)	Y				✓		✓	✓	✓	✓		✓	✓		✓	✓	✓
Beckmann (2005)	Y[a]															✓	
Bennett & Nelson (2004)	Y				✓		✓	✓	✓	✓	✓	✓			✓		✓
Billstein et al. (2004)	Y			✓	✓		✓	✓	✓	✓		✓		✓	✓		✓
Darken (2003)	Y		✓				✓	✓	✓	✓				✓		✓	✓
Jensen (2003)	Y						✓										
Jones et al. (2000)	N																
Long & DeTemple (2003)	Y	Y[b]			✓	✓		✓	✓	✓				✓	✓	✓	✓
Masingila et al. (2002)	Y[a]	Y		✓	✓		✓	✓		✓				✓	✓		
Musser et al. (2003)	Y			✓	✓		✓	✓		✓		✓		✓	✓		✓
O'Daffer et al. (2002)	Y	Y			✓		✓	✓	✓	✓					✓		✓
Parker & Baldridge (2004)	Y					✓			✓		✓					✓	
Sonnabend (2004)	Y						✓	✓	✓	✓				✓	✓		✓
Wheeler & Wheeler (2005)	Y			✓	✓		✓	✓	✓	✓		✓			✓		✓
Total	13	3	1	4	8	2	10	10	9	10	2	5	1	6	9	5	9

[a]For one volume only. [b]A mathematical lexicon showing etymology of common mathematical terms.

[1] At least one book (Krause, 1991), recently out of print, has a very long history as a textbook for such classes, growing out of a book first published in the "New Math" era (Brumfiel & Krause, 1968).

[2] In other research, the authors of this paper are studying the characteristics of courses and instructors in mathematics classes for elementary teachers. For more information see http://www.educ.msu.edu/Meet/.

[3] In another part of our research, we are planning an analysis of textbooks over time, tracking how editions change as related to policy or other external pressures.