ME ET The Mathematical Education of Elementary Teachers

August, 2006

Dear Instructor,

Research on teaching elementary mathematics has exposed an urgent need for strengthening the mathematical knowledge of elementary teachers. Yet there is little systematic knowledge about the content and organization of mathematics courses taken by pre-service elementary teachers. Understanding what prospective elementary teachers are learning about mathematics is vital to improving the teaching of mathematics in our nation's schools.

As a college mathematics instructor, you are being asked to participate in a research study funded by the National Science Foundation focusing on mathematics courses for pre-service elementary teachers. Your participation is important as it will help us gain an accurate picture of what is taught and learned by prospective teachers in their mathematics content courses. Instructors in a select number of institutions are being asked to participate. Although the course you teach might not be designated specifically for prospective elementary teachers, it has been identified as one of the most common for prospective elementary teachers to take at your institution.

Your participation would include three components: a student survey given early in the semester; a follow-up survey for students late in the semester; and a survey for you to complete late in the semester. The first survey is pre-test of the students in your class, administered during the first or second week of class. It is designed to measure aspects of their mathematical knowledge when they begin your course along with some demographic information and items about their attitudes and beliefs about mathematics. The second component is a survey given in the last week of your class, designed to capture growth in your students' mathematical knowledge. Both the surveys are important to represent the gain in mathematical content knowledge prospective elementary teachers experience as a result of their coursework. The final component is a survey that you complete late in the semester about the organization and content of your course. This survey is necessary because it allows us to connect particular elements of mathematics courses with what students actually learn. The student surveys take about 25 minutes and 20 minutes respectively; the survey you complete takes about 45 minutes.

We realize that participation involves considerable time on your part. There are several direct benefits for you. First, we will prepare a report of your students' performance on the pre-test within two weeks of receiving the completed pre-tests. This data can provide you with valuable data on your students' mathematical knowledge. For example, you might find that your students have strengths or weaknesses in particular areas. This can help you more effectively target your instruction to their academic needs. This report will also compare the performance of your students to other students in our study, so you can gauge their relative knowledge. An example of a report on the pretest we can provide to you is enclosed.

Most significantly, this research can provide you with direct evidence about your teaching, which can be valuable for hiring, tenure, and promotion considerations. After the post-test and survey have been received, we will prepare a final report that shows the gains your students made during your course. This report will compare the gains of your students compared to gains of students in similar courses at other institutions. It is rare that instructors have this type of data on their teaching outcomes. Having direct evidence on your students' learning can serve as evidence to a hiring or tenure committee that you take your teaching seriously and are committed to student learning.

The final report we prepare for you will also highlight elements of mathematics courses that lead to greater student learning gains. It is likely that these reports will be available prior to any publications that emerge from this study. Thus, you will have access to research that can inform your own teaching. Demonstrating how research on effective teaching and course organization affects your teaching can, in turn, be used as additional evidence of your commitment to teaching.

In addition to these direct benefits to you, your participation would contribute to our knowledge of pre-service teachers' mathematical knowledge and results of this study can help improve elementary teachers' mathematical knowledge, which can, in turn, improve the mathematics achievement of elementary students. At this time of intense political and educational focus on mathematics education, institutions of higher education and funding agencies are pushing to see more cooperation and collaboration between education and mathematics faculties. This research presents an opportunity for you as a mathematician to participate in a mathematics education project. Your input will contribute to mathematics educators' learning about what you do, while the feedback we give you and your inside view of the methods of the study will contribute to your understanding of mathematics education research.

Finally, in appreciation of your participation in the study, we will give you a gift certificate for $25 to Barnes & Noble to buy materials that will help with your teaching; and a second gift certificate for $15 upon completion of your survey. Your students will also receive a small token of our appreciation – a pen or pencil with a mathematics logo – for each of the surveys they complete.

You can be assured of complete confidentiality. Your identity and individual responses will not be reported in any report from this research. We will send your course report only to you, so that you can choose how to use this information. Further, if asked about the participation of any individual, we will neither confirm nor deny that individual instructors participated in our research. Although you are asked to provide your name and section number, this information will never be reported. Your name and section number will only be used to connect your responses to your students' responses. Once data collection is complete, your name and identifying information will be deleted.

I would be happy to answer any questions you may have. Please don't hesitate to email or call me. My email address is mccrory@msu.edu. My phone number is 517-353-8565. Thank you for your consideration.

Sincerely,

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ATT: Sample Report

Pretest Results from the Survey of Mathematics Classes for Elementary Teachers

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(SAMPLE REPORT. The data in this report are not real, although they are based on pilot data. The actual report provided to instructors will use data collected in the Fall, 2006, study.)

This report presents results for your section of the *Survey of Mathematics for Elementary Teachers* conducted in the first week of your mathematics course. The survey consisted of three parts: 1) mathematics content knowledge, 2) attitudes and beliefs about mathematics, and 3) demographic and mathematical background. This report presents summary information about your students for each of these areas. Results both from your course and from courses at other institutions in the study are included so you can compare your students to students in similar courses. The population of courses included in this study is the first mathematics course that prospective elementary teachers usually take in colleges and universities with teacher certification programs.

Section 1: Mathematical Content Knowledge Measures

The survey administered to your students included a measure of the mathematical content knowledge necessary for teaching elementary mathematics. In particular, this measure focused on the knowledge of number and operations in elementary mathematics. This measure was developed and extensively tested by a team at the University of Michigan-Ann Arbor specifically to measure the mathematical knowledge of elementary teachers.[1]

Table 1 presents a measure of your students' average scores, called 'theta', on the mathematical content knowledge measure. This table also reports the average score for all similar courses in the study. The scores were calculated using a two-parameter item response theory model and are scale scores ranging from negative infinity to positive infinity. The scores tell a person's ability or location on the underlying continuum of the ability being measured, in this case, mathematical content knowledge. Theta is scaled so that larger values correspond to more of the underlying trait. The scale is basically ordinal, and does not constitute an interval scale. That is, the difference between 0 and 1 might be different from the difference between 0 and -1. However, when the ability in the population can be reasonably assumed to be normally distributed, it is possible to interpret the scores as if they are z-scores because the scores are standardized scores with mean of zero and standard deviation of 1 as z-scores. Because it is reasonable to assume a normal distribution of ability in mathematical content knowledge for the prospective elementary teacher

[1] For more information on this measure, see Ball, Bass, & Hill (2003) and Hill, Schilling, & Ball (2004b). See also Hill, Rowan, & Ball (2004a) for how the measure relates to elementary student achievement.

students, it is possible to compare the average performance of your students with the average performance of other classes in the study as if the scores are z-scores. At the end of the semester, gain scores in theta metric for your class as well as for your institution and other institutions in the study will be provided based on the post-test results. This will give you an opportunity to see your students gain from the class and to compare your class to other classes and institutions.

Table 1: Summary statistics for mathematical content knowledge*

Item	N	Mean theta	Std Dev	Minimum	Maximum
Your course					
All students	30	-.048	1.1	-2.241	2.105
Students in bottom 10%	3	-1.57	1.2	-2.241	-.220
Students in top 10%	3	1.12	0.3	.011	2.105
All participating courses in the study					
All students	1000	-0.000	1.12	-3.010	2.552
Students in bottom 10%	100	-1.730	1.19	-3.010	-.220
Students in top 10%	100	1.02	1.27	-.012	2.552

*Non-response is treated as ‘incorrect’ in this report.

(These are simulated data and do not reflect results from a real class.)

Released Items

The items in this test are taken from the Learning Mathematics for Teaching (LMT) item bank (Hill et al., 2004b). In general, to maintain the integrity of the test, items from LMT cannot be released publicly. Some sample items have been released, however, three of which were used on this survey.[2] Results on these items for your course and for all participating courses in the study are shown in Table 2.

(Here, we give a two sample items. On the actual report, we will include all the released items that are given to your students in the fall.)

Table 2: Results for released items, your course and all courses

(simulated data)

	Your course		All courses	
Item	**N**	**% Correct**	**N**	**% Correct**
2	30	80	1000	62
8	30	53	1000	58

The released items are shown below, along with the responses of your course and all participating courses.

[2] Measures copyright 2005, Study of Instructional Improvement (SII)/Learning Mathematics for Teaching/Consortium for Policy Research in Education (CPRE). Not for reproduction or use without written consent of LMT. Measures development supported by NSF grants REC-9979873, REC- 0207649, EHR-0233456 & EHR 0335411, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

2. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

Table 3: Response frequency by option, Item 2

(simulated data)

		Percent	
Item	Answer	Your course	All courses
A	**1**	**33**	**31**
	2	47	51
	3	20	18
B	**1**	**42**	**39**
	2	43	42
	3	15	19
C	**1**	**67**	**64**
	2	29	31
	3	4	5

Correct response shaded.

8. As Mr. Callahan was reviewing his students' work from the day's lesson on multiplication, he noticed that Todd had invented an algorithm that was different from the one taught in class. Todd's work looked like this:

```
  983
 x  6
  488
+5410
 5898
```

What is Todd doing here? (Mark ONE answer.)

a) Todd is regrouping ("carrying") tens and ones, but his work does not record the regrouping.

b) Todd is using the traditional multiplication algorithm but working from left to right.

c) Todd has developed a method for keeping track of place value in the answer that is different from the conventional algorithm.

d) Todd is not doing anything systematic. He just got lucky – what he has done here will not work in most cases.

Table 4: Response frequency by option, Item 8

(simulated data)

	Percent	
Answer	Your course	All courses
a	23	25
b	17	12
c	**53**	**58**
d	7	4
No response	0	1

Correct response shaded.

Section 2: Attitudes and Beliefs Measures

There were 20 items on attitudes and beliefs about mathematics in the survey. The items were on a 5 point scale with 1=strongly disagree and 5=strongly agree. The items consisted of 4 factors with reliabilities (Cronbach's alpha) shown in table 6, determined in statistical analyses in the pilot study. The four factors measure beliefs about 1) usefulness of mathematics, 2) multiple ways of doing mathematics, 3) the nature of mathematics, and 4) processes of doing mathematics. All items composing the whole measures are presented in the Appendix.

Table 6. Attitudes and beliefs factors, all institutions

(NOTE: The data in this table are taken from the pilot study and are actual results from surveys of 198 students.)

Factors	**N**	**Mean**	**Std Dev**	**Cronbach's Alpha**	**Minimum**	**Maximum**
Usefulness	198	3.75	.65	0.75	1.60	5.00
Multiple ways	198	2.64	.40	0.85	1.67	4.33
Nature of math	198	3.54	.50	0.70	2.17	5.00
Process	198	3.18	.64	0.65	1.50	4.50

The first factor, "usefulness", consists of six items which are reported in Appendix A. This refers to the belief that mathematics is a useful enterprise. The mean score of 3.75 indicates that respondents overall agree to the view that mathematics is useful. The second factor, 'multiple ways' indicates the views about whether mathematics problems can be solved in many different ways. The mean score 2.64 indicates that responding students tend to think of mathematics as having a single way of solving a problem. The "nature of mathematics" factor measures students ideas about the rigor and precision of mathematics. The mean score of 3.54 indicates that responding students agree with this view of mathematics. The last factor, process, refers to students beliefs about their own experience with mathematics, whether they enjoy doing mathematics, whether they see it as a creative endeavor. The mean score of 3.18 for 'process' factor suggests that this group of students, on average, likes mathematics and sees it as creative. The large standard deviation, however, suggests that students vary considerable on this measure, as well as on "usefulness."

Table 7 reports Pearson's correlations among the four 'attitudes and beliefs' factors and performance on the test, part 2 of the survey. For each factor, the first row reports correlation coefficients, and the second row reports the degree of probability of rejecting the hypothesis that the correlation is not statistically significant. As shown from the correlations in Table 7, 'usefulness' and 'process' have a moderate positive correlation (r=.491) indicating that students tend to think mathematics is useful when they have positive experiences about mathematics or vice versa. The negative correlation between 'multiple ways' and 'usefulness' suggests that students tend to think mathematics is useful when they disagree with the view of mathematics as having a single way of solving a problem and vice versa. Content knowledge has positive correlations with both ‘usefulness’ (r=.318) and ‘process’ (r=.379) and negative correlation with ‘multiple ways’ (-.166). This suggests that students with more content knowledge tend to view mathematics as useful and creative and as having multiple ways of solving a problem. This is an interesting finding that needs verification through additional research, which this study aims at.

Table 7. Pearson Correlations of Attitudes and Beliefs Factors and Math Achievements

(Actual data from the pilot study)

	Usefulness	Multiple ways	Nature of math	Process	Content Knowledge
Usefulness	1	-.206*	.196*	.491*	.318*
		.003	.005	.000	.000
Multiple ways		1	.106	-.131	-.166*
			.128	.059	.16
Nature of math			1	.000	.063
				.996	.365
Process				1	.379*
					.000
Content Knowledge					1

* significant at the α = .05 level.

Section 3. Background Information

This section reports the demographic and mathematical background characteristics of students in your course. These results include data on your course, and for all participating courses in the study.

Demographics

Table 8. Personal information

(simulated data)

		Your course		All Courses	
		Frequency	Percent	Frequency	Percent
Gender	Male	2	6.7	80	7.1
	Female	28	93.3	1050	92.9
College Year	Freshman	12	40.0	320	28.3
	Sophomore	8	26.7	440	38.9
	Junior	6	20.0	240	21.2
	Senior	4	13.3	110	9.8
	Post BA	0	0.0	20	1.8
Major field of study	Child Development	2	6.7	30	2.7
	Education	24	80.0	1080	95.6
	Special Education	3	10.0	10	0.88
	Mathematics	1	3.3	10	0.88
Number of college mathematics courses to date*	1	14	46.7	380	33.6
	2	13	43.3	300	26.6
	3	3	10.0	210	18.6
	4 or more	0	0.0	240	21.2

*Includes courses in statistics and computer sciences

Achievement in Mathematics

Table 9 reports distribution of ACT mathematics scores. Across the entire population, more than one-fifth of the respondents who took the ACT did not report their ACT mathematics score. Therefore, scores are reported for 83 students. The mode score is 29 with 12 students, and mean score is 25.31 as shown in tables 10 and 10. Table 10 also reports standards deviation, minimum, and maximum scores.

(We are also collecting SAT data and will report the scores for the test taken by the majority of students in your class.)

Table 9. ACT scores

(simulated data)

	Your class		All classes	
ACT Score	Frequency	Percent	Frequency	Percent
Did not take ACT	2	6.67	50	5.0
17	1	3.33	10	1.0
18	1	3.33	33	3.3
19	2	6.67	47	4.7
21	1	3.33	90	9.0
22	2	6.67	40	4.0
23	2	6.67	50	5.0
24	3	10.00	87	8.7
25	2	6.67	63	6.3
26	2	6.67	60	6.0
27	2	6.67	50	5.0
28	1	3.33	98	9.8
29	2	6.67	122	12.2
30	2	6.67	30	3.0
32	2	6.67	22	2.2
34	1	3.33	18	1.8
36	1	3.33	10	1.0
No Response	1	3.33	120	12.0

Table 10. ACT score statistics

(simulated data)

	N	Mean	Std Dev	Minimum	Maximum
Your course	30	25.63	4.88	17	36
All courses	1000	25.31	4.97	17	36

Table 11 reports Pearson's correlations between the students' ACT mathematics score and their content knowledge score on this test. As expected, ACT score has a statistically significant relationship with students' mathematical content knowledge.

Table 11. Pearson correlation coefficients

(simulated data)

	Correlation between ACT and Mathematical Content Knowledge
Your course	**0.52**
	<.001
All courses	**0.49**
	<.001

High School Mathematics Courses

Table 12 reports the mathematics courses taken by your students in high school. The frequency reports the number of students who took that course in at least one grade in high school. The percent taking each course was computed excluding the non-respondents to the course. There is a small difference in the number of total respondents across the courses (105 to 112) Most of the students took algebra I (95 %), algebra II (90 %), and geometry (95 %). About two-thirds of the respondents took pre-calculus, but only 27 % took calculus in high schools.

Table 12: High school mathematics courses

(simulated data)

	Your course		All courses	
High School Math Courses	**Frequency**	**Percent**	**Frequency**	**Percent**
Basic or general mathematics	23	77	870	87
Tech-prep, business, consumer, or other applied mathematics	8	28	210	21
Intro to algebra or pre-algebra	26	87	920	92
Algebra I	10	32	950	95
Geometry	28	95	950	95
Algebra II with or without trigonometry	27	90	900	90
Trigonometry (as a separate course)	13	42	420	42
Pre-calculus	20	67	670	67
Unified, integrated, or sequential mathematics	4	14	140	14
Probability or statistics	7	22	220	22
Calculus	8	27	270	27
Discrete or finite mathematics	2	6	60	6
Other mathematics courses	2	8	80	8
Computer programming	5	16	160	16

References

Ball, D. L., Bass, H., & Hill, H. C. (2003). *Knowing and using mathematical knowledge in teaching: Learning what matters*. Ann Arbor: University of Michigan.

Hill, H. C., Rowan, B., & Ball, D. L. (2004a). *Effects of teachers' mathematical knowledge for teaching on student achievement*. Ann Arbor: University of Michigan.

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Appendix: Attitudes and Beliefs Items

Factor 1: Usefulness

1. Mathematics is useful for solving everyday problems.
2. Almost all people use mathematics in their jobs.
3. If one engages in mathematical tasks, one can discover new things (connections, rules, concepts).
4. Mathematics entails a fundamental benefit for society.
5. Mathematics is useful for every profession.

Factor 2: Multiple Ways

1. There is only one correct way to solve a mathematics problem.
2. Everything important about mathematics is already known by mathematicians.
3. Math problems can be done correctly in only one way.
4. Usually there is more than one way to solve mathematical tasks and problems.

Factor 3: Nature of mathematics

1. Mathematics is a collection of rules and procedures, which prescribe how to solve a task.
2. Mathematical thought is characterized by abstraction and logic.
3. Hallmarks of mathematics are clarity, precision and unambiguousness.
4. Mathematics involves remembering and application of definitions, formulas, mathematical facts and procedures.
5. Mathematics is characterized by rigor, namely rigor of definition and rigor of formal mathematical argumentation.
6. To do mathematics demands much practice, correct application of routines, and problem solving strategies.

Factor 4: Process

1. I'm good at mathematics.
2. I like mathematics.
3. Learning mathematics is mostly memorizing facts.
4. In mathematics you can be creative.
5. To solve math problems you have to be taught the right procedure.
6. In mathematics many things can be discovered and tried out by oneself.