STUDY OF INSTRUCTIONAL IMPROVEMENT/
LEARNING MATHEMATICS FOR TEACHING

CONTENT KNOWLEDGE FOR TEACHING MATHEMATICS MEASURES (CKT-M MEASURES)

MATHEMATICS RELEASED ITEMS
2005

University of Michigan, Ann Arbor
610 E. University #1600
Ann Arbor, MI 48109-1259
(734) 647-5233
www.soe.umich.edu/lmt

 Measures development supported by NSF grants REC-9979873, REC- 0207649, EHR-0233456 & EHR 0335411, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

Study of Instructional Improvement

lmt learning mathematics for teaching project

April, 2005

Dear Colleague:

Thank you for your interest in our survey items measuring teachers' knowledge for teaching mathematics. Because of the expense of developing and piloting items, we do not release items from our general pool over the web. Instead, we provide here a small set of items that illustrate our larger item pool; those interested in using this larger item pool can contact Geoffrey Phelps (gphelps@umich.edu) for information about training sessions and permissions.

These released items may be useful as open-ended prompts which allow for the exploration of teachers' reasoning, as materials for professional development or teacher education, or as exemplars of the kinds of mathematics teachers must know to teach. We encourage their use in such contexts. However, this particular set of items is, as a group, NOT appropriate for use as an overall measure, or scale, representing teacher knowledge. In other words, one cannot calculate a teacher score that reliably indicates either level of content knowledge or growth over time.

We ask users to keep in mind that these items represent steps in the process of developing measures. Many of these released items failed, statistically speaking, in our piloting; in these cases, items may contain small mathematical ambiguities or other imperfections. If you have comments or ideas about these items, please feel free to contact one of us by email at the addresses below.

These items are the result of years of thought and development, including both qualitative investigations of the content teachers use to teach elementary mathematics, and quantitative field trials with large numbers of survey items and participating teachers. Because of the intellectual effort put into these items by SII investigators, we ask that *all* users of these items satisfy the following requirements:

1) Please request permission from SII/LMT for any use of these items. To do so, contact Geoffrey Phelps at gphelps@umich.edu. Include a brief description of how you plan to use the items, and if applicable, what written products might result.

2) In any publications, grant proposals, or other written work which results from use of these items, please cite the development efforts which took place at SII/LMT by referencing this document:

> Hill, H.C., Schilling, S.G., & Ball, D.L. (2004) Developing measures of teachers' mathematics knowledge for teaching. Elementary School Journal 105, 11-30.

3) Refrain from using these items in multiple choice format to evaluate teacher content knowledge in any way (e.g., by calculating number correct for any individual teacher, or gauging growth over time). Use in professional development, as open-ended prompts, or as examples of the kinds of knowledge teachers might need to know is permissible.

You can also check the SII website (http://www.sii.soe.umich.edu/) or LMT website (http://www.soe.umich.edu/lmt) for more information about this effort.

Below, we present three types of released item – elementary content knowledge, elementary knowledge of students and content, and middle school content knowledge. Again, thank you for your interest in these items.

Sincerely,

Deborah Loewenberg Ball
Study of Instructional Improvement
University of Michigan
dball@umich.edu

Heather Hill
Learning Mathematics for Teaching
University of Michigan
hhill@umich.edu

Study of Instructional Improvement/Learning Mathematics for Teawching

Content Knowledge for Teaching Mathematics Measures (CKT-M measures)

Released Items, 2005

ELEMENTARY CONTENT KNOWLEDGE ITEMS

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	Yes	No	I'm not sure
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3
c) The number 8 can be written as 008.	1	2	3

2. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

<table>
<tr><th>Student A</th><th>Student B</th><th>Student C</th></tr>
<tr><td>35
x 25
125
+75
875</td><td>35
x25
175
+700
875</td><td>35
x 25
25
150
100
+600
875</td></tr>
</table>

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

3. Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

a) Four is an even number, and odd numbers are not divisible by even numbers.

b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).

c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.

d) It only works when the sum of the last two digits is an even number.

4. Ms. Chambreaux's students are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

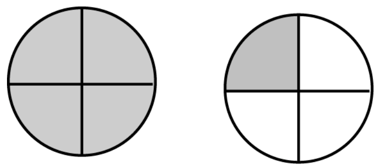
a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.

b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.

c) Check to see whether 371 is divisible by any prime number less than 20.

d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

5. Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)



a) 5/4

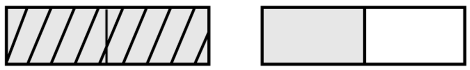
b) 5/3

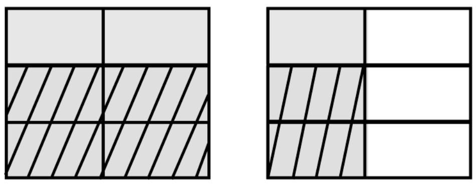
c) 5/8

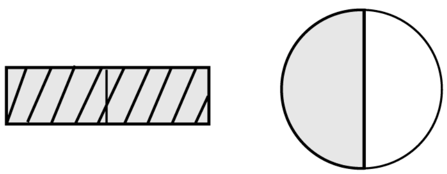
d) 1/4

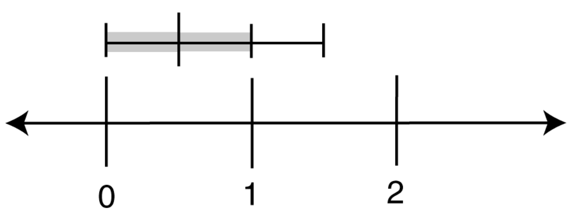
6. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately.

Which model below cannot be used to show that $1\frac{1}{2} \times \frac{2}{3} = 1$? (Mark ONE answer.)

A) 

B) 

C) 

D) 

7. Which of the following story problems could be used to illustrate $1\frac{1}{4}$ divided by $\frac{1}{2}$? (Mark YES, NO, or I'M NOT SURE for each possibility.)

	Yes	No	I'm not sure
a) You want to split $1\frac{1}{4}$ pies evenly between two families. How much should each family get?	1	2	3
b) You have $1.25 and may soon double your money. How much money would you end up with?	1	2	3
c) You are making some homemade taffy and the recipe calls for $1\frac{1}{4}$ cups of butter. How many sticks of butter (each stick = $\frac{1}{2}$ cup) will you need?	1	2	3

8. As Mr. Callahan was reviewing his students' work from the day's lesson on multiplication, he noticed that Todd had invented an algorithm that was different from the one taught in class. Todd's work looked like this:

$$\begin{array}{r} 983 \\ \underline{\times\ \ 6} \\ 488 \\ +\underline{5410} \\ 5898 \end{array}$$

What is Todd doing here? (Mark ONE answer.)

a) Todd is regrouping ("carrying") tens and ones, but his work does not record the regrouping.

b) Todd is using the traditional multiplication algorithm but working from left to right.

c) Todd has developed a method for keeping track of place value in the answer that is different from the conventional algorithm.

d) Todd is not doing anything systematic. He just got lucky – what he has done here will not work in most cases.

ELEMENTARY KNOWLEDGE OF STUDENTS AND CONTENT ITEMS

9. Mr. Garrett's students were working on strategies for finding the answers to multiplication problems. Which of the following strategies would you expect to see some elementary school students using to find the answer to 8 x 8? (Mark YES, NO, or I'M NOT SURE for each strategy.)

	Yes	No	I'm not sure
a) They might multiply 8 x 4 = 32 and then double that by doing 32 x 2 = 64.	1	2	3
b) They might multiply 10 x 10 = 100 and then subtract 36 to get 64.	1	2	3
c) They might multiply 8 x 10 = 80 and then subtract 8 x 2 from 80: 80 – 16 = 64.	1	2	3
d) They might multiply 8 x 5 = 40 and then count up by 8's: 48, 56, 64.	1	2	3

10. Students in Mr. Hayes' class have been working on putting decimals in order. Three students — Andy, Clara, and Keisha — presented 1.1, 12, 48, 102, 31.3, .676 as decimals ordered from least to greatest. What error are these students making? (Mark ONE answer.)

a) They are ignoring place value.

b) They are ignoring the decimal point.

c) They are guessing.

d) They have forgotten their numbers between 0 and 1.

e) They are making all of the above errors.

11. You are working individually with Bonny, and you ask her to count out 23 checkers, which she does successfully. You then ask her to show you how many checkers are represented by the 3 in 23, and she counts out 3 checkers. Then you ask her to show you how many checkers are represented by the 2 in 23, and she counts out 2 checkers. What problem is Bonny having here? (Mark ONE answer.)

a) Bonny doesn't know how large 23 is.

b) Bonny thinks that 2 and 20 are the same.

c) Bonny doesn't understand the meaning of the places in the numeral 23.

d) All of the above.

12. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students focused on particular difficulties that they are having with adding columns of numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

$$\text{I)}\quad \begin{array}{r} {\scriptstyle 1} \\ 38 \\ 49 \\ +\,65 \\ \hline 142 \end{array} \qquad \text{II)}\quad \begin{array}{r} {\scriptstyle 1} \\ 45 \\ 37 \\ +\,29 \\ \hline 101 \end{array} \qquad \text{III)}\quad \begin{array}{r} {\scriptstyle 1} \\ 32 \\ 14 \\ +\,19 \\ \hline 64 \end{array}$$

Which have the same kind of error? (Mark ONE answer.)

a) I and II

b) I and III

c) II and III

d) I, II, and III

13. Ms. Walker's class was working on finding patterns on the 100's chart. A student, LaShantee, noticed an interesting pattern. She said that if you draw a plus sign like the one shown below, the sum of the numbers in the vertical line of the plus sign equals the sum of the numbers in the horizontal line of the plus sign (i.e., 22 + 32 + 42 = 31 + 32 + 33). Which of the following student explanations shows sufficient understanding of <u>why this is true</u> for all similar plus signs? (Mark YES, NO or I'M NOT SURE for each one.)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

	Yes	No	I'm not sure
a) The average of the three vertical numbers equals the average of the three horizontal numbers.	1	2	3
b) Both pieces of the plus sign add up to 96.	1	2	3
c) No matter where the plus sign is, both pieces of the plus sign add up to three times the middle number.	1	2	3
d) The vertical numbers are 10 less and 10 more than the middle number.	1	2	3

14. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students around particular difficulties that they are having with subtracting from large whole numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

I	II	III
4 12	4 15	6 9 8 15
~~5~~0~~2~~	3~~5~~00~~5~~	~~7~~~~0~~~~0~~~~5~~
− 6	− 6	− 7
406	34009	6988

Which have the same kind of error? (Mark ONE answer.)

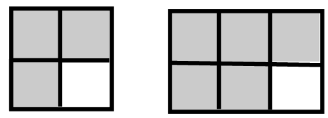
a) I and II

b) I and III

c) II and III

d) I, II, and III

15. Takeem's teacher asks him to make a drawing to compare $\frac{3}{4}$ and $\frac{5}{6}$. He draws the following:



and claims that $\frac{3}{4}$ and $\frac{5}{6}$ are the same amount. What is the <u>most likely</u> explanation for Takeem's answer? (Mark ONE answer.)

a) Takeem is noticing that each figure leaves one square unshaded.

b) Takeem has not yet learned the procedure for finding common denominators.

c) Takeem is adding 2 to both the numerator and denominator of $\frac{3}{4}$, and he sees that that equals $\frac{5}{6}$.

d) All of the above are equally likely.

16. A number is called "abundant" if the sum of its proper factors exceeds the number. For example, 12 is abundant because 1 + 2 + 3 + 4 + 6 > 12. On a homework assignment, a student incorrectly recorded that the numbers 9 and 25 were abundant. What are the most likely reason(s) for this student's confusion? (Mark YES, NO or I'M NOT SURE for each.)

	Yes	No	I'm not sure
a) The student may be adding incorrectly.	1	2	3
b) The student may be reversing the definition, thinking that a number is "abundant" if the number exceeds the sum of its proper factors.	1	2	3
c) The student may be including the number itself in the list of factors, confusing proper factors with factors.	1	2	3
d) The student may think that "abundant" is another name for square numbers.	1	2	3

MIDDLE SCHOOL CONTENT KNOWLEDGE ITEMS

17. Students sometimes remember only part of a rule. They might say, for instance, “two negatives make a positive.” For each operation listed, decide whether the statement “two negatives make a positive” sometimes works, always works, or never works. (Mark SOMETIMES, ALWAYS, NEVER, or I’M NOT SURE)

	Sometimes works	Always works	Never works	I’m not sure
a) Addition	1	2	3	4
b) Subtraction	1	2	3	4
c) Multiplication	1	2	3	4
d) Division	1	2	3	4

18. Mrs. Smith is looking through her textbook for problems and solution methods that draw on the distributive property as their primary justification. Which of these familiar situations could she use to demonstrate the distributive property of multiplication over addition [i.e., a (b + c) = ab + ac]? (Mark APPLIES, DOES NOT APPLY, or I’M NOT SURE for each.)

	Applies	Does not apply	I’m not sure
a) Adding $\frac{3}{4}+\frac{5}{4}$	1	2	3
b) Solving 2x – 5 = 8 for x	1	2	3
c) Combining like terms in the expression $3x^2 + 4y + 2x^2 - 6y$	1	2	3
d) Adding 34 + 25 using this method: $\begin{array}{r} 34 \\ \underline{+25} \\ 59 \end{array}$	1	2	3

19. Students in Mr. Carson's class were learning to verify the equivalence of expressions. He asked his class to explain why the expressions a – (b + c) and a – b – c are equivalent. Some of the answers given by students are listed below.

Which of the following statements comes <u>closest</u> to explaining why a – (b + c) and a – b – c are equivalent? (Mark ONE answer.)

a) They're the same because we know that a – (b + c) doesn't equal a – b + c, so it must equal a – b – c.

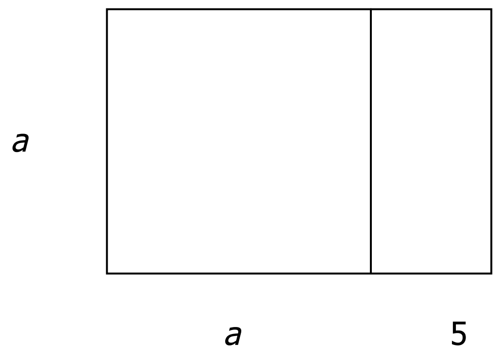
b) They're equivalent because if you substitute in numbers, like a=10, b=2, and c=5, then you get 3 for both expressions.

c) They're equal because of the associative property. We know that a – (b + c) equals (a – b) – c which equals a – b – c.

d) They're equivalent because what you do to one side you must always do to the other.

e) They're the same because of the distributive property. Multiplying (b + c) by –1 produces –b – c.

20. Ms. Whitley was surprised when her students wrote many different expressions to represent the area of the figure below. She wanted to make sure that she did not mark as incorrect any that were actually right. For each of the following expressions, decide whether the expression correctly represents or does not correctly represent the area of the figure. (Mark REPRESENTS, DOES NOT REPRESENT, or I'M NOT SURE for each.)



		Correctly represents	Does not correctly represent	I'm not sure
a)	$a^2 + 5$	1	2	3
b)	$(a + 5)^2$	1	2	3
c)	$a^2 + 5a$	1	2	3
d)	$(a + 5)a$	1	2	3
e)	$2a + 5$	1	2	3
f)	$4a + 10$	1	2	3

21. Ms. Hurlburt was teaching a lesson on solving problems with an inequality in them. She assigned the following problem.

$$-x < 9$$

Marcie solved this problem by reversing the inequality sign when dividing by – 1, so that $x > -9$. Another student asked why one reverses the inequality when dividing by a negative number; Ms. Hurlburt asked the other students to explain. Which student gave the <u>best explanation</u> of why this method works? (Mark ONE answer.)

a) Because the opposite of x is less than 9.

b) Because to solve this, you add a positive x to both sides of the inequality.

c) Because $-x < 9$ cannot be graphed on a number line, we divide by the negative sign and reverse the inequality.

d) Because this method is a shortcut for moving both the x and 9 across the inequality. This gives the same answer as Marcie's, but in different form: $-9 < x$.