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The ME.ET Project: Overview and Update

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Abstract

The Mathematical Education of Elementary Teachers (MEET) project is an NSF funded study of undergraduate mathematics classes for prospective elementary teachers in two states – Michigan and South Carolina – and in New York City. MEET researchers are investigating undergraduate mathematics classes required for elementary certification. We have collected data from mathematics departments at 57 institutions and piloted and administered over 2000 assessment forms to students at 13 schools in 58 sections of mathematics classes of 43 instructors. We have also analyzed all the published textbooks written specifically for such classes. Our data include information about who teaches the classes, what topics are covered, and what students learn. Initial analyses suggest that these classes are little influenced by policy or certification testing, but may be strongly influenced by the textbook used. Instructors are not familiar with key documents such as the CBMS publication *Mathematical Education of Teachers.* Results from all parts of the study will be discussed in other papers in the symposium.

Project Overview

The problem of teachers' mathematical knowledge has been a subject of research and policy for many years. While it is almost a truism to say that K-6 teachers' mathematical knowledge is inadequate, the problem is more complex than the simple assertion suggests. Fundamental questions of what mathematical knowledge is needed and how it can be taught and learned continue to demand attention, as improvement in K-6 *students'* mathematical achievement in the US lags in national and international assessments (Braswell, Daane, & Grigg, 2003; Braswell et al., 2001; Sherman, Honegger, McGivern, & Lemke, 2003). Recent policy documents (Committee on Science and Mathematics Teacher Preparation, 2000; Committee on the Mathematical Education of Teachers, 1991; Conference Board of the Mathematical Sciences, 2001; Leitzel, 1991; Mathematical Sciences Education Board, 1996; Mathematical Sciences Education Board & National Research Council, 2001; National Research Council & Committee on Science and Mathematics Teacher Preparation, 2001; RAND Mathematics Study Panel, 2002) urge continuing research on teachers' mathematical knowledge and propose agendas to address this issue. This project focuses on 1) opportunities to learn mathematics provided in undergraduate teacher preparation programs for K-6 teachers; 2) prospective teachers' learning given those opportunities; and 3) how those opportunities relate to research and policy on teachers' mathematical knowledge.

The mathematics community has a long history of analyzing and discussing what mathematics teachers need to know. In recent years, a Mathematical Association of America report (Leitzel, 1991) made recommendations about topics that should be included in teachers' education. The 2001 CBMS report "augments those recommendations, by giving more attention to the mathematical conceptions of K–12 students and how their teachers can be better prepared to address these ideas," (CBMS 2001, p. 13). The report suggests areas of study and desired outcomes, describes in some detail aspects of knowledge in each of several topical areas and grade level groups, and makes eleven recommendations. While acknowledging that quality is more important than quantity, the CBMS report recommends,

> i. Prospective elementary grade teachers should be required to take at least 9 semester-hours on fundamental ideas of elementary school mathematics.
>
> ii. Prospective meddle grades teachers of mathematics should be required to take at least 21 semester-hour of mathematics, that includes at least 12 semester-hours on fundamental ideas of school mathematics appropriate for middle grades teachers. (Recommendation 2, p. 8).

In the last few years, researchers have investigated the mathematics that teachers use in practice (Ball & Bass, 2000; Ferrini-Mundy, Burrill, Floden, & Sandow, 2003), and teacher educators have designed mathematics courses specifically for prospective high school mathematics teachers (Usiskin, 2000; Usiskin, Peressini, Marchisotto, & Stanley, 2002). Ball and Bass (2003) present convincing evidence that mathematical content knowledge for teaching is more complex than might be immediately apparent. They suggest that teachers need mathematical knowledge that, while clearly mathematical and not pedagogical, is nonetheless knowledge that math majors and even professional mathematicians may not have. For example, knowing how to do multidigit multiplication with borrowing (e.g., 35 x 25) is *mathematical knowledge* that most adults have. Knowing how to reverse engineer that problem to understand how the incorrect answers 245, 1055, or 105 might be derived is *mathematical knowledge for teaching*. Each of those answers has a logical explanation linked to a misuse or misunderstanding of mathematics, and, Ball argues, is mathematical even though it would not be part of the repertoire of a typical undergraduate mathematics course (Ball, NPR Broadcast on May 28, 2004). Hill, Rowan, and Ball (2004) name these *common* (mathematics that is taught and learned in the course of a good secondary or postsecondary education) and *specialized* (knowledge distinct to the repertoire of a teacher) content knowledge for teaching.

In this study, we use these ideas, as well as items developed by Hill, Ball and colleagues, to investigate the mathematics taught and learned in undergraduate mathematics classes for future elementary teachers.

Research Design

The project includes three major strands: (a) Investigating *opportunities to learn* currently provided to prospective elementary teachers by analyzing textbooks, surveying instructors, and gathering other data about courses in undergraduate mathematics; (b) Investigating *preservice teacher learning* in and through these courses; (c) Investigating the coherence of courses with respect to research and policy related to the undergraduate mathematical education of prospective K-6 teachers. In this paper, and in others from this project, we refer to the preservice teachers as students, since they are the students in the classes we study. If we refer to K-12 students, they will be clearly designated as such.

The project framework is illustrated in Figure 1. Briefly, we aim to understand what is offered in the first required class for prospective elementary teachers by examining textbooks, questions addressed to instructors about content, goals, and teaching methods in the class, and by general questions to the department chair about course design. The data also include information about how the courses are organized and implemented: who teaches them, how instructors are selected, who is responsible for assessment, and more. Finally, we analyze national, state, and local policies and policy instruments (e.g., certification tests) to assess alignment between policy and courses, including asking instructors about their familiarity with key policy documents.

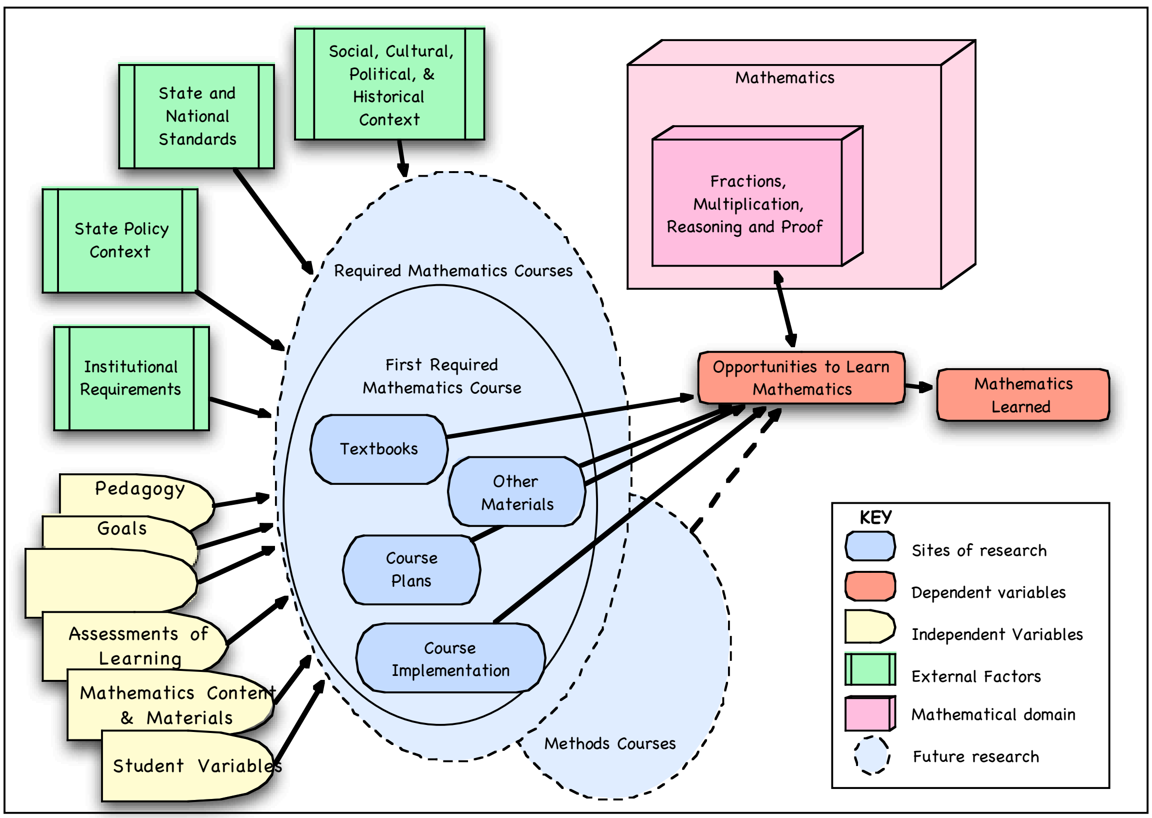


Figure 1: ME.ET Conceptual Map

Site selection

We identified states that vary with respect to policy and outcomes in mathematics education. In particular, we used Title II reports, the annual *Quality Counts* report from *Education Week*, National Assessment of Educational Progress (NAEP) data, and published research on teacher education and policy to compare states across a number of dimensions. In the end, we selected Michigan, South Carolina, and New York

City as our sites. Table 1 give summary data for the three states. Our reasons for selecting them are explained briefly below.

Michigan

Although Michigan's mathematics NAEP scores have historically been above average, they have been weaker in recent years, and the "achievement gap" problem in Michigan has remained acute. In the *Quality Counts* survey, Michigan received low scores for efforts to improve teacher quality.

South Carolina

In contrast to Michigan, South Carolina has historically had below average NAEP scores, but in recent years, their scores have risen dramatically, and the "achievement gap" has narrowed significantly. South Carolina is among the highest scorers in the *Quality Counts* survey of efforts to improve teacher quality. Recent policy initiatives in South Carolina – including incentives for National Board certification and an extensive teacher induction program – contribute to a policy environment that is innovative and strong.

New York City

New York state has historically had above average outcomes on NAEP mathematics test and a reputation for rigorous student testing and high standards for teacher education. In the *Quality Counts* report, NY scored B-, between SC and MI. In the end, we selected New York City because of other research on teacher education at NYCity institutions, the Pathways project which has followed students into their induction years. That project did not collect detailed data about the content of teacher preparation and we thought the two projects could provide a more comprehensive view when taken together.

In the end, we have 70 institutions in three locations covering the spectrum of institutions of higher education: large and small, public and private, religious, historically black.

Table 1

Information by State: Certification, Tests, Achievement

	MI	NY	SC
Certification: Number of certifying Institutions	32	118	31
Number of NCATE[1] certified institutions	16	25	22
Number of TEAC[2] certified institutions	3	24	0
Testing: Praxis required?	no	no	yes
PRAXIS Pass Rate	NA	NA	90%
State Test Required?	yes	yes	no
Quality Counts (QC) K-12 Standards grade[3]	B+	A	A
QC Teacher Quality Improvement Score[3]	66	81	92
New certifications from in-state[4]	7641	32128	2049
New certifications from out of State[4]	977	0	1514
Percent out of state	11%	0%	42%
NAEP Information[3,4]			
%Proficient and above (Math) 4th grade, NAEP 2003	38%	36%	36%
%Proficient State Test	65%	78%	34%
Difference, NAEP-State	-31	-45	2

	MI	NY	SC
NAEP 4th Math (mean, US public 237)	238	238	238
Mean for White students (US public 246)	245	247	250
Mean for Black students (US public 220)	211	222	223
NAEP 8th Math (mean, US = 278)	277	280	281
Mean for White students (US public 286)	285	290	294
Mean for Black students (US public 257)	247	259	263
Quality Counts Info[3]			
Middle School: Major or Minor in Math	minor[6]	minor	Minor
Student teaching -- min wks	6	8	12
Standards aligned with test	Y	Y	Y
Teacher Prep Accountability Process	No	No	Yes

[1]National Council for Accreditation of Teacher Education. Date from NCATE Web site, www.ncate.org

[2] Techer Education Accreditation Council. Data from TEAC Web site, www.teac.org

[3] From Title II Web site

[4] From NCES Web site, 2005 data

[5] From Ed Weekly Quality Counts 2005 Web site

[6] Michigan requires a subject area major or 3 minors for elementary education majors

Content focus

Although our larger goal is to understand the range of mathematics offered to prospective elementary teachers in their undergraduate programs, we needed to focus our research on a smaller subset of that mathematics to make the project feasible. We focus in two ways: First, we look in depth only at the first class students are required to take for elementary certification not at every mathematics class offered. This decision is practical, but also justified because it makes a detailed comparison across institutions possible. We gather more general information about other courses at the department level, but go in greater detail for the first required class.

Second, within that class, we focus on three big areas of the elementary curriculum: fractions, multiplication, and reasoning and proof. Our questions are about opportunity to learn (OTL), and research on OTL suggests that it is a construct best operationalized at a detailed, fine-grained level (Floden, 2002; McDonnell, 1995). The instructor survey, student assessment, and textbook analysis focus on these three areas.

Why these topics? All three are important in elementary mathematics and they represent different kinds of knowledge. Study of multiplication starts early and continues throughout elementary school and into the middle years. Children learn, and teachers teach, multiplication of every kind of number, and multiplication of algebraic expressions. As one of the four operations that are a major focus of the elementary curriculum, multiplication has a unique position across the elementary curriculum. Fractions, our second topic, is a kind of number known to be a stumbling block for many children and teachers. It is key to later mathematics, truly fundamental to learning and doing algebra (Wu, 1999). Reasoning and proof is a basic part of doing mathematics, known to be foundational and important yet routinely downplayed or left to implicit learning. Many students falter when they encounter their first mathematics courses that requires proofs, whether it is a traditional high school geometry class or an undergraduate class much later in their schooling. Thus, in these three topics, we have an operation, a kind of number, and a way of thinking, spanning the elementary curriculum temporally and intellectually.

As with narrowing our focus to a single course, the focus on three areas is not carried out with blinders on: we ask questions about other topics, analyze the textbooks for total coverage, and in other ways, include a broader range of mathematics. When we assess OTL, however, we work with these three topics. The logic is that what we find for these three will provide much needed insights into these courses, and the methods we develop can be used to study other topics and other courses in future work.

Instrumentation

We have developed and piloted 4 instruments, used for data collection in 2006-7. These are:

1. Department survey
2. Instructor survey with subsections:
    i. Course information
    ii. Personal information
    iii. Opportunity to learn measure
3. Student assessment (4 forms) with subsections:
    i. Attitudes and beliefs scales
    ii. Mathematics knowledge scale
    iii. Personal information
4. Textook analysis protocols

Papers in the symposium address the design of these instruments and results from each element of the project.

Data Collection

One of the major efforts during the first year of the project was obtaining Institutional Review Board (IRB) approvals at all 70 institutions in the study. After obtaining approval at Michigan State University, we learned that many institutions nonetheless require a complete review with their own forms and protocols. We now have separate approval at 20 institutions, and are still in the application process at the remaining 6 schools. The remaining institutions accepted the MSU approval. Our low response rate in NYC is entirely explained by complicated IRB procedures that have delayed approvals.

Department survey

From May through November 2006, we contacted and surveyed departments at all eligible schools in the three locations. 54 schools completed the survey, a phone interview conducted by one of the project researchers. The interview covered basic information about the classes offered for elementary teachers, including identifying instructors by name. We built our instructor sample through the department survey.

School selection

At a subset of institutions, we are administering pre- and post-tests to students in the first required mathematics class. To select schools for participation in this part of the study, we first chose those that graduate the largest number of elementary education students in each state, supplemented by schools that added to the diversity of our sample (e.g., historically black, small religious schools). We used data from the US News annual report on higher education to adjust our selection to reflect a range of school characteristics. We chose only schools that offer at least two sections of the first mathematics class in a given year, yielding a list of 33 schools for inclusion in this part of the study. We developed a list of 30 schools (see Table 1 for a breakdown by state). Of the 30 in the original sample, 20 had complete IRB approval by August 15, 2006. Of these, two declined to participate, one requested participation in a later semester, and the others did not respond. Our final sample for fall 2006 included 13 schools.

In the 13 schools, we identified 45 instructors who were teaching the first mathematics class for elementary education students in fall 2006. Of these, 28 agreeed to participate. Because of the low response rate and the problems we had with IRB and contacts in NYC, we will have another round of data collection in fall 2007.

Site	Schools					Instructors		
	Total	Selected	Eligible[a]	Pre-post	Rate[b]	Eligible[c]	Pre-post	Rate
MI	30	14	11	7	64%	27	20	74%
SC	26	10	9	6	67%	18	8	44%
NYC	14	6	1	0	0%	NA	NA	NA
Total	70	30	10	13	65%	45	28	62%

[a]Eligible if IRB's completed by August 15, 2006.

[b]Rate of participation = Pre-post/(Selected– No IRB). The overall rate without taking IRB eligibility into account (Pre-post/Selected) is 43%.

[c]Every instructor at the participating schools was asked to participate. This number is the total number of instructors identified by the departments at the partcipating schools.

[d]The course coordinators at the single institution in NYC where we had IRB approval decided not to participate and asked us not to contact their instructors to administer the pre- post-tests. This school will be replaced in fall 2007. The targeted class was a general mathematics class, the first required for elementary education certification, but not targeted for that population. The instructors felt the tests would be discouraging for their population. Unfortunately, we have no student data so far from schools that require only general mathematics classes for elementary teachers.

Pre- and Post-tests

As described in the paper by Kim in this symposium, we developed pretest and posttest forms to assess student (preservice teacher) gains. Our forms include three parts: a survey of attitudes and beliefs about mathematics; a mathematics test; and a survey of personal information. The mathematics test uses items from the Learning Mathematics for Teachers (LMT) project at the University of Michigan. Using these items, we piloted several forms in 2005-6 and created our final scales using both the item data from LMT and our own pilot data. In the end, we have four forms–two pretests and two posttests–based on two disjoint sets of 20 items each and 6 common items used for equating the forms. This design allows us to give entirely different tests for pre- and post- to each class, while continuing to accumulate equating data as our sample grows.

We include measures of attitudes and beliefs because of the widespread, although not well-establised, notion that preservice teachers' ideas about mathematics impact their learning. Although this is not a study of attitudes and beliefs or of changes in attitudes and beliefs, we wanted to be able to control for basic attitudes toward mathematics as we sought explanations for gains over the semester. It seemed logical that students who "hate" mathematics might learn less than those who "love" it, and that this could be an important factor. Our items are based on prior research (cf., Fennema, 1976; Schoenfeld, 1989), relying heavily on a recent survey from the international Preliminary Teacher Education Development Study (P-TEDS) project which was able to measure differences in attitudes and beliefs about mathematics across countries (Blömeke, 2005; Blömeke & Felbrich, 2004). Their items were based on work by Grigrutsch (1998). Information about the development of these scales will be available in a technical report on our Web site.

Table 2

Form information for pilot, pretest and postests

Purpose	Form	Items			Students	
		Mathematics	Attitudes and Beliefs	Personal Info	Total	Matched Pairs
Pilot Pretest	A, C, D	44	43	Yes	221	150
Pilot Posttest	B, C, D	44	43	Yes	291	With A, C, D
Pretest	E	26 (set 1 + common)	21	Yes	400	300
	F	26 (set 2 + common)	21	Yes	423	314
Posttest	G	20 (set 2)	No	Yes[a]	430	With E
	H	20 (set 1)	No	Yes[a]	333	With F
Totals					2098[b]	764

[a]The personal information section was included on posttest for students who did not take the pretest. We did not include attitudes and beliefs on the posttest because those responses may have been influenced by the class and we wanted only comparable data. In addition, we are using the attitudes and beliefs data only as an independent variable in gain score models, which would not include students who did not take the pretest.

[b]This is the number of forms administered. The total number of individual students who participated is: 1334: 764 matched and 570 unmatched.

Instructor survey

In December 2006, surveys were sent to 137 instructors identified in the department survey at all 57 institutions from which we had responses. In spite of our efforts to encourage participation, as of April 2007, we have received 55 surveys, a participation rate of 40%. 49 of the responses are included in the data reported in other papers. We are continuing to work on getting a higher response rate and analyzing these data. The survey protocol is available on our Web site.

Textbook analysis protocols

Over the last 4 years, our research group has analyzed textbooks written for undergraduate mathematics classes for elementary teachers. When this part of the work started, there were 21 such books in print and several others that were self-published or being written, often by instructors who were dissatisfied with the books available. Since then, many of the books have gone out of print, while others have been published. Today, we have a list of 14 books in print, with one more that we know of about to be published, and another in preparation.

Our analyses have gone through many phases, as described in a paper by Siedel in this symposium. The effort has been to understand and compare the mathematical entailments of the books, not to evaluate or rank them. Our position with respect to the quality of the books is that instructors have different reasons and methods for using textbooks and thus there is no gold standard that can be applied to judge books. On the other hand, we assert that these books need to be mathematically sound and on that basis – in the case where incorrect or confusing mathematics is presented – evaluation is necessary.

We are still developing ways to quantify opportunity to learn (OTL) provided by textbooks, aiming to include data from the textbook analysis in our statistical models. The instructor survey includes information about what chapters of the book are used and from this, combined with our analysis, we hope to be able to create one or more variables that capture OTL from textbook use. This has proven to be a very difficult task.

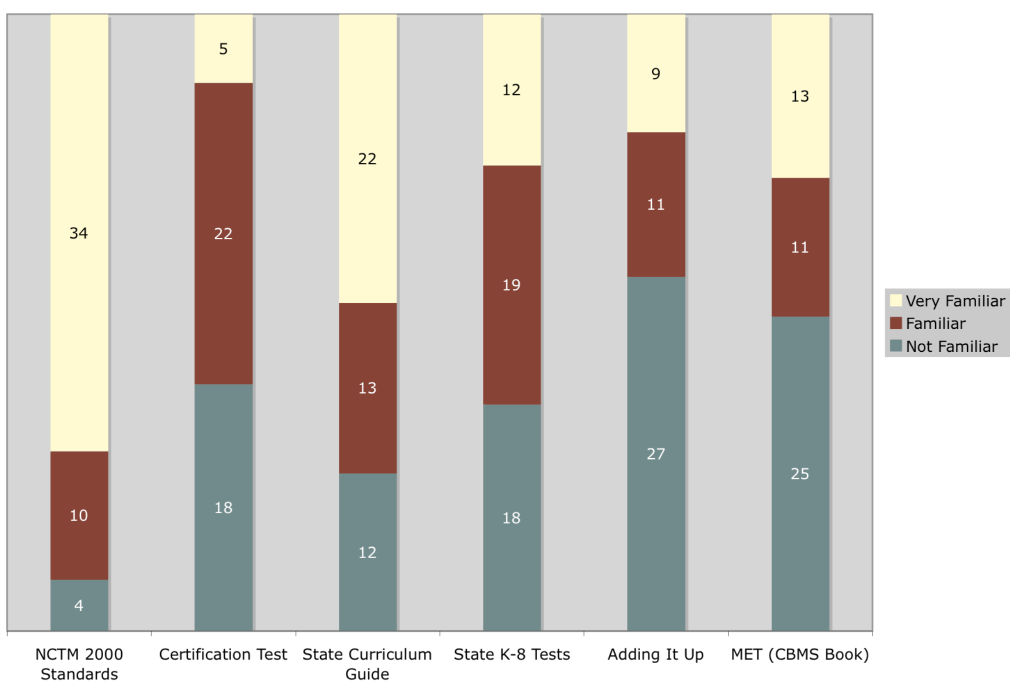
Policies and tests for teacher certification

A final element of our study is an analysis of policies related to teacher certification and tests used as critera for certification. Our question here is how policy and high stakes tests impact mathematics classes for teachers. We chose the three states because of the differences in their approach to policies that impact teacher quality, so we expect to see some differences in how policies are manifest in classes for teachers. At the same time, we hypothesize that links between policy and classes will be weak, as will links between tests and classes. There is very little research – none that we have found – that assesses alignment between content classes for teachers, certification policies, and certification tests. We expect that little is known in part because the link has historically been weak.

Our research so far suggests that our initial hunches are correct. The three states have very few policies that are explicit about the mathematics content teachers need to know. Certification tests in all three states appear to be a weak instrument for regulating or determining what teachers learn in part because they are easy (high pass rates) but also because information about what is on them is sparse. It would be hard for an instructor to "teach to the test" because so little information is available that specifies what should be taught. In addition, we found in our instructor survey that instructors are not familiar with the tests or with other relevant policy documents. Figure 1 shows the number of instructors reporting that they are not familiar, familiar, or very familiar with some key policy documents. As the figure shows, the most familiar document is the NCTM Standards (National Council of Teachers of Mathematics, 2000) followed by the state curriculum guides. Less familiar documents include *Adding it Up* (Kilpatrick, Swafford, & Findell, 2001), and the *Mathematical Education of Teachers* (Conference Board of the Mathematical Sciences (CBMS), 2001). A surprisingly large proportion of these instructors – 18 out of 49, or 37% – were not familiar with their own state's certification test (PRAXIS II for SC, MTTC for Michigan. NY not yet included in these data), while only 5 out of 49 (10%) reported being very familiar with the test.

Figure 1

Instructor familiarity with tests and policy documents (n=49)



The instructor survey also asks how instructors use various resources in their teaching, another way to consider the impact of policy documents. Figure 2 shows how the primary textbook is used by instructors. It is interesting that 5 of the 49 do not use the primary textbook at all. Possibly the book is mandated by the department but the instructor designs the class without reference to the book. Figure 3 shows how a number of other resources are used by instructors in the sample. Congruent with their level of familiarity, the most used document is the NCTM Standards, followed by other textbooks different from the primary text. Certification tests are infrequently used by instructors, further suggesting that their influence on the content of courses is small.

Finally, Figure 4 shows how many instructors reported not using a particular document for any of the purposes we asked about. Out of 49 instructors who responded to the survey, 34 do not use certification tests at all and 28 do not use state standards for teacher education. These data suggest that, with the possible exception of the NCTM document, standards, tests, and policies are not a strong influence on mathematics classes for teachers.

Figure 2

Instructor use of primay textbook by type of use (n=49)

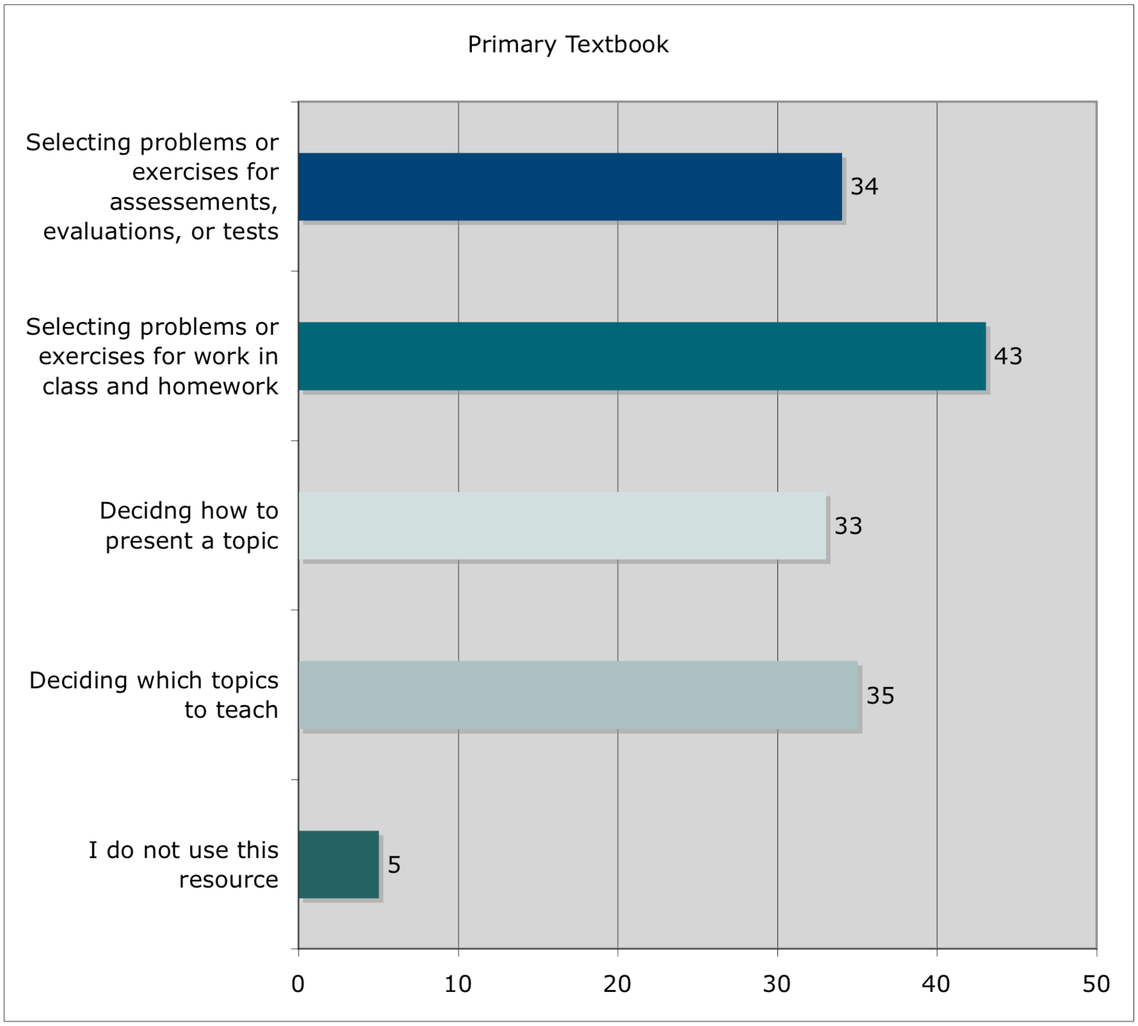


Figure 3

Instructor use of resource, by resource and type of use (n=49)

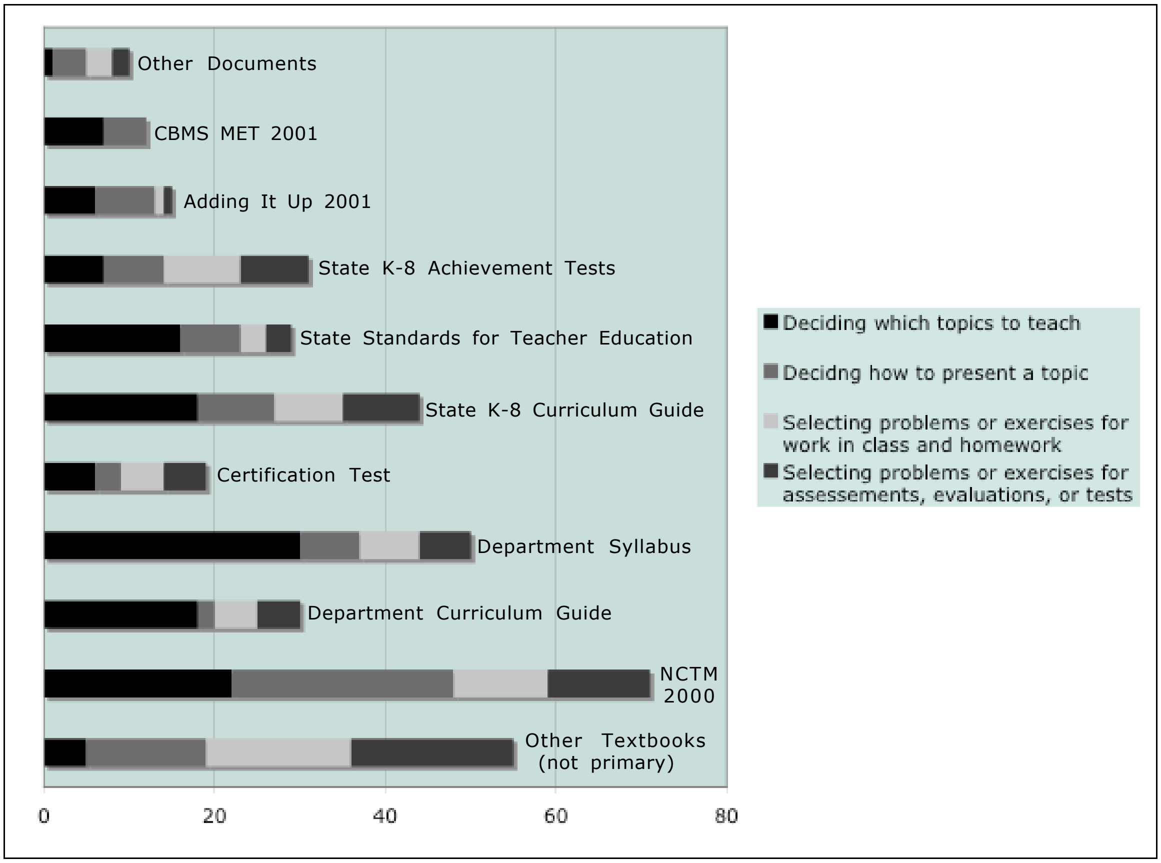
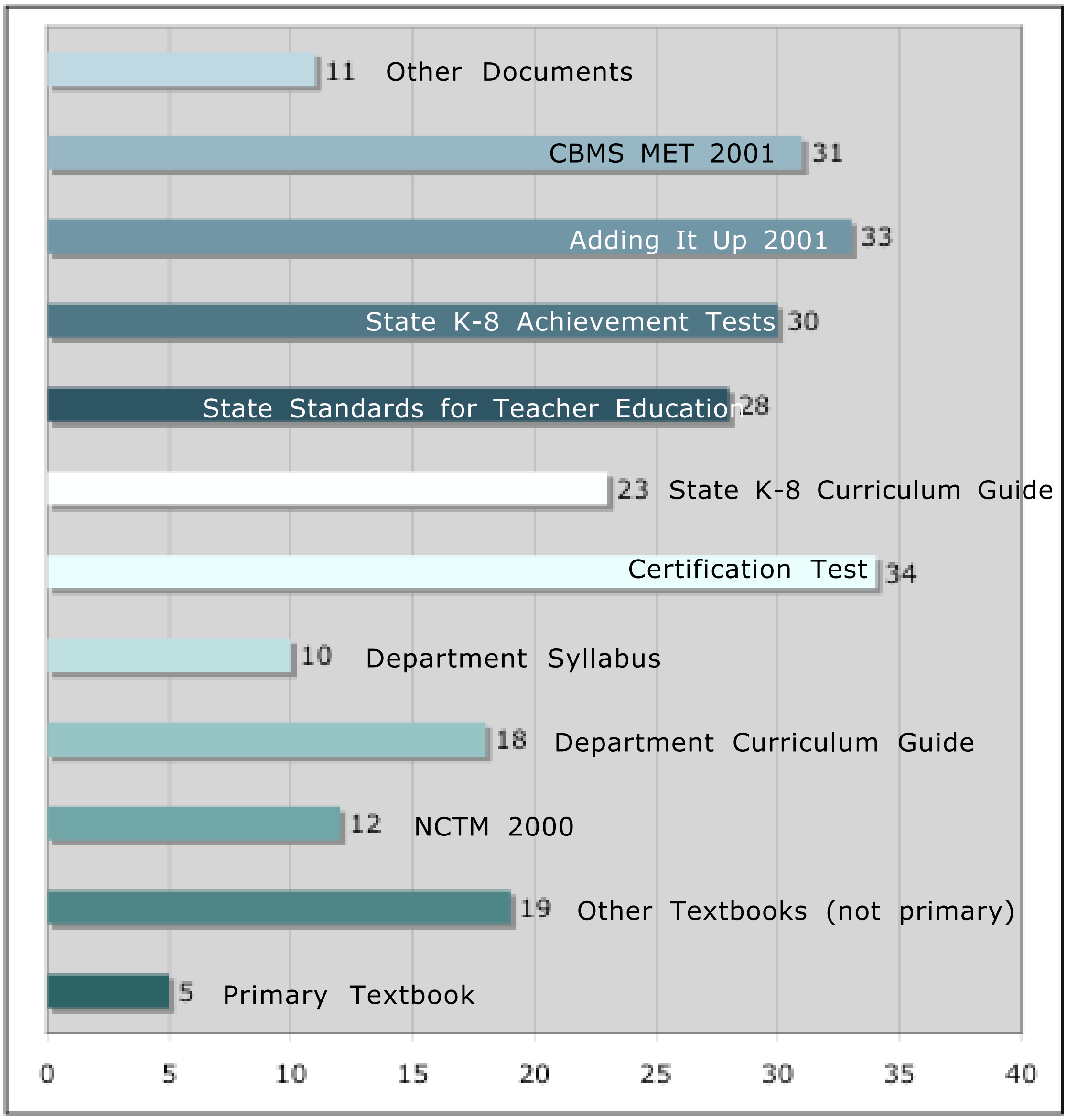


Figure 4

*Instructors who **do not** use a given resource for any purpose (n=49)*



Conclusions and Next Steps

Our data collection will continue in the 2007-8 academic year. We will survey mathematics departments in NY, collect additional student pre/post data in all three states, and contact instructors to complete the instructor survey. Our goal is to have about 1500 matched pairs of students in the pre/post data from at least 60 instructors.

We hope to have over 100 instructors complete the instructor survey, out of a population we estimate at 250 (all instructors who teach these classes in SC, MI, and NYC).

Once data collection is complete, we will be able to report more extensive data at every level of the research, and most importantly, we will be able to construct multilevel models to investigate opportunity to learn and its impact on student learning.

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