CAREER: Knowledge for teaching mathematics: The impact of mathematics courses on prospective elementary teachers’ mathematical knowledge.

Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach. The mathematical knowledge needed by teachers at all levels is substantial, yet quite different from that required by students pursuing other mathematics-related professions. Prospective teachers need to understand the fundamental principles that underlie school mathematics, so that they can teach it to diverse groups of students as a coherent, reasoned activity and communicate an appreciation of the elegance and power of the subject. (Recommendation 1, CBMS Conference Board of the Mathematical Sciences, 2001, p. 7)

In light of this recommendation, and others in the CBMS report, do courses for prospective teachers offer them what they need? Are these courses successful? That is, do prospective teachers learn what they need to know? These are the driving question of the research proposed here.

The problem of teachers’ mathematical knowledge has been a subject of research and policy for many years. While it is almost a truism to say that K-6 teachers’ mathematical knowledge is inadequate, the problem is more complex than the simple assertion suggests. Fundamental questions of what mathematical knowledge is needed and how it can be taught and learned continue to demand attention, as improvement in K-6 students’ mathematical achievement in the US lags in national and international assessments (Braswell, Daane, & Grigg, 2003; Braswell et al., 2001; Sherman, Honegger, McGivern, & Lemke, 2003). Recent policy documents (Committee on Science and Mathematics Teacher Preparation, 2000; Committee on the Mathematical Education of Teachers, 1991; CBMS 2001; Leitzel, 1991; Mathematical Sciences Education Board, 1996; Mathematical Sciences Education Board & National Research Council, 2001; National Research Council & Committee on Science and Mathematics Teacher Preparation, 2001; RAND Mathematics Study Panel, 2002) urge continuing research on teachers’ mathematical knowledge and propose agendas to address this issue. The research proposed here will focus on 1) opportunities to learn mathematics provided in undergraduate teacher preparation programs for K-6 teachers; 2) prospective teachers’ learning given those opportunities; and 3) how those opportunities relate to research and policy on teachers’ mathematical knowledge.

Since Shulman’s seminal presidential address to AERA (1986), research on teachers’ subject matter knowledge has escalated in every domain. Shulman and colleagues (Grossman, 1990; Shulman, 1987; Wilson, Shulman, & Richert, 1987; Wineburg & Wilson, 1988) identified three kinds of knowledge of importance for teaching: content knowledge, pedagogical content knowledge (PCK), and general pedagogical knowledge. Subject matter knowledge included the kind of knowledge a professional in the field might have, at a level appropriate for teaching. PCK included knowledge of representations that are likely to help students learn the content and of typical student misconceptions and errors. General pedagogical knowledge included aspects of teaching not specifically related to a subject, such as organizing and managing group work. In mathematics, this division has turned out to be less crisp than the labels might suggest, as the nature and substance of subject matter knowledge for teaching has come under scrutiny.
Ball and colleagues (Ball, Lubienski, & Mewborn, 2001) review of research on teacher knowledge in mathematics outlines the problematic nature of its conceptualization and study. It points out that early studies of teacher knowledge used teacher characteristics, primarily the number of mathematics courses taken, as a proxy for mathematical knowledge. These studies produced mixed results, suggesting that, while some number of mathematics courses seems to correlate with student outcomes, there is a point at which returns are diminished or even negative. At that ill-defined point, more courses do not result in higher student achievement (p. 442-3). A report by Wilson, Floden, and Ferrini-Mundy (2001; 2002) indicates that rigorous research about teacher preparation is sparse, both across disciplines and within the field of mathematics. In the few studies that met the requirements to be included in their review, they too found mixed results about the development of mathematical knowledge. In one study, the number of mathematics methods courses teachers took had larger correlations with K-12 pupil outcomes than the number of mathematics courses while in other studies methods courses did not appear to have the same effects (Monk & King, 1994, cited in Wilson et al, 2001). In a recent study, Greenberg and colleagues (Greenberg, Rhodes, Ye, & Stancavage, 2004) used NAEP data to investigate the relationship between teachers’ certification status and eighth grade student achievement. They found that certification status correlated with student achievement, even when controlling for ability, race, and socio-economic status. None of these studies, however, considers how mathematics courses or other coursework, or experiences required for certification, change what teachers know or do to achieve better outcomes.

The mathematics community has a long history of analyzing and discussing what mathematics teachers need to know. In recent years, the Mathematical Association of America report (Leitzel, 1991) made recommendations about topics that should be included in teachers’ education. The recent CBMS report (2001) “augments those recommendations, by giving more attention to the mathematical conceptions of K–12 students and how their teachers can be better prepared to address these ideas,” (p. 13). The report suggests areas of study and desired outcomes, describes in some detail aspects of knowledge in each of several topical areas and grade level groups, and makes eleven recommendations. While acknowledging that quality is more important than quantity, the CBMS report recommends that “prospective elementary grade teachers should be required to take at least 9 semester-hours on fundamental ideas of elementary school mathematics,” (Recommendation 2, p. 8).

In the last few years, researchers have investigated the mathematics that teachers use in practice (Ball & Bass, 2000; Ferrini-Mundy, Burrill, Floden, & Sandow, 2003), and teacher educators have designed mathematics courses specifically for prospective high school mathematics teachers (Usiskin, 2000; Usiskin, Peressini, Marchisotto, & Stanley, 2002). Ball and Bass (2003) present convincing evidence that mathematical content knowledge for teaching is more complex than might be immediately apparent. They suggest that teachers need mathematical knowledge that, while clearly mathematical and not pedagogical, is nonetheless knowledge that math majors and even professional mathematicians may not have. For example, knowing how to do multidigit multiplication with borrowing (e.g., 35 x 25) is mathematical knowledge that most adults have. Knowing how to reverse engineer that problem to understand how the incorrect answers 245, 1055, or 105 might be derived is mathematical knowledge for teaching. Each of those answers has a logical explanation linked to a misuse or misunderstanding of mathematics, and, Ball argues, is mathematical even though it would not be part of the repertoire of a typical undergraduate mathematics
course (Ball, NPR Broadcast on May 28, 2004). Hill, Rowan, and Ball (2004) name these common (mathematics that is taught and learned in the course of a good secondary or postsecondary education) and specialized (knowledge distinct to the repertoire of a teacher) content knowledge for teaching.

Over the last two decades, there have been a number of studies of preservice teachers’ knowledge of mathematics, often aimed at particular topics or processes (Ball, 1990; Borko et al., 1992; Eisenhart et al., 1993; Even, 1990; Graeber, Tirosh, & Glover, 1989; Ma, 1998; Simon, 1993; Simon & Blume, 1994, 1996; Tirosh, Fischbein, Graeber, & Wilson, 1999; M. Wilson, 1994). In general, these studies indicate that although they may be able to do the mathematics of elementary school and obtain correct answers, preservice teachers often cannot give adequate explanations, provide representations of ideas, or generate illustrative problems. Their understanding of elementary mathematics appears to be quite weak, and this is true even for students who have more extensive mathematics coursework than might be typical of elementary education majors (Borko et al., 1992). Ma (1998) contrasts the knowledge of elementary teachers in China and the US, and identifies the difference as “profound understanding of fundamental mathematics.”

Research Objectives

In the research proposed here, the purposes are to gain insights into mathematical knowledge for teaching and to understand how undergraduate courses can contribute to improving the mathematical education of elementary teachers. The research questions are: What opportunities to learn are provided to prospective elementary teachers in their undergraduate mathematics education, and what do they learn? How do teachers’ opportunities to learn mathematics relate to research and policy on teacher knowledge? The research will have three primary areas: (a) Investigating opportunities to learn currently provided to prospective elementary teachers through textbooks, instructors, and courses in undergraduate mathematics; (b) Investigating teacher learning in and through these courses; (c) Investigating the coherence of courses with respect to research and policy related to the undergraduate mathematical education of prospective K-6 teachers.

Building on recent conceptions and theories of mathematical knowledge for teaching, I will investigate these constructs in three lines of work: (a) analysis of textbooks, curriculum materials and policy documents; (b) survey research aimed at mathematics departments and instructors in selected states; and (c) qualitative case studies at institutions in those states, including assessments of teacher learning. The objectives of the research are both basic and applied, seeking to increase theoretical understanding of teachers’ mathematical knowledge through the lens of those who teach them mathematics and create materials for use in that teaching; and to provide data useful in guiding policy, standards, and practice in the undergraduate mathematics education of K-6 teachers.

The hypotheses of the research are: (a) the undergraduate mathematics education of prospective teachers impacts their learning of mathematics; (b) mathematics courses for prospective elementary teachers typically teach general mathematical knowledge and less frequently attend to specialized mathematical knowledge for teaching; (c) there is little consistency across courses and textbooks in what is taught to prospective elementary teachers, and little agreement about what should be taught; and (d) teachers’ knowledge of mathematics for teaching increases when they are provided with opportunities that specifically address mathematical knowledge for teaching.
One obvious caveat to this research is that prospective teachers have opportunities to learn mathematics outside of mathematics courses for teachers: mathematics methods courses, mathematics courses other than those designed for teachers, and field (student teaching) experiences are all possible sites for learning mathematics. This study will focus only on the mathematics courses aimed at prospective teachers.  

The first part of the research has already begun. I have identified 18 textbooks currently in print or in preparation (see reference list), developed a framework for their analysis, and completed preliminary analysis of the books. I selected three topics for in-depth analysis: fractions, multiplication, and reasoning and proof. The rationale for this selection is explained below, along with some of the preliminary findings. 

Relationship to the State of Knowledge in the Field

The proposed research draws on several bodies of literature including studies of teachers’ mathematical knowledge outlined above, studies of the learning of mathematics, and textbook analysis. The latter are reviewed briefly below. 

Studies of mathematics learning. Over the last three decades, there has been considerable research about how children learn mathematics (Kilpatrick, Swafford, & Findell, 2001). On the topics of fractions, multiplication, and reasoning and proof, focal for this study, researchers have investigated student learning including where children have difficulty and common misconceptions or errors (Barnett, Goldenstein, & Jackson, 1994; Behr, Harel, Post, & Lesh, 1993; Bransford, Brown, & Cocking, 1999; Carpenter, Fennema, & Romberg, 1993; Chazan, 1993; Even, 1990; Fischbein, Deri, Nello, & Marino, 1985; Fischbein & Kedem, 1982; Greer, 1993; Hanna, 1995; Harel & Confrey, 1994; Kamii & Clark, 1995; Kilpatrick et al., 2001; Knuth, 2002; Lampert, 1986; Mack, 1995; Martin & Harel, 1989; Senk, 1985; Wu, 1999). A few studies focus on preservice teachers (Graeber et al., 1989; Martin & Harel, 1989; Schram, Wilcox, Lanier, & Lappan, 1988; Simon & Blume, 1996). The research provides a basis for understanding what mathematics teachers are likely to encounter in K-6 classrooms, with insights into particular problems that could call on specialized mathematical knowledge. For example, research on children’s learning of multiplication suggests that children learn to expect multiplication to “make bigger” (Fischbein et al., 1985), making the transition to operations with fractions and decimals difficult. Translating into mathematical knowledge for teaching means creating or recognizing problems and representations of multiplication that do not reinforce the “multiplication makes bigger” stereotype. Thus, the research on learning is a source for hypotheses about specialized mathematics that could be useful to teachers. The CBMS (2001) report suggests that this kind of knowledge does not clearly fall into either mathematics or pedagogy and may be neglected by both mathematics and methods courses. Research on mathematics learning also suggests what kinds of problems prospective teachers may have in their own learning in undergraduate mathematics courses. 

Studies of textbooks. Another aspect of the proposed research is a study of undergraduate mathematics textbooks. Other studies of mathematics textbooks have focused on textbooks and curricula for K-6 students, investigating their relationship to standards (Kulm, 1999; Kulm & Grier, 1998; Senk & Thompson, 2003; Stake & Easley, 1978); how they influence what is taught and learned in K-12 classrooms (Elliott & Woodward, 1990; Freeman & Porter, 1989; Porter & Freeman, 1989; Sosniak & Perlman, 1990; Stodolsky, 1989; Venezky, 1992); how teachers themselves learn from their pupils’ textbooks and curriculum materials (Ball & Cohen, 1996; Ball & Feiman-Nemser, 1988; Collopy, 1999;
and how materials compare across international boundaries (Carter, Li, & Ferruci, 1997; Schmidt, McKnight, & Raizen, 1997; Schmidt, McKnight, Valverde, Houang, & Wiley, 1997; Stigler, Fuson, Ham, & Kim, 1986). These studies use a range of techniques on which I will draw to analyze and compare books. At MSU, the Teacher Education Study in Mathematics, TEDS-M, is beginning an international study of curriculum in mathematics education that will include undergraduate textbooks. I will coordinate with Dr. Jack Schwille, the PI for the TEDS-M project, to share methods and data.

What is Missing?

There is little research that explores what prospective K-6 teachers have an opportunity to learn, in terms of mathematical topics, processes, or dispositions. Research about undergraduate mathematics education for these students has focused on outcomes. I have found no research that addresses the question of the content of their undergraduate mathematics education or whether there are patterns across institutions. Although the mathematics community has provided much guidance for designing these courses, we do not know if their recommendations are being followed.

Some scholars have suggested that the content of these courses is “obvious.” You just teach them what they have to teach their pupils. Recent research on teacher knowledge cited above, along with my preliminary analysis of the textbooks written for such courses make plain that the content is far from obvious. Although there is some consistency in what is included in textbooks (whole numbers, fractions, decimals, operations, etc.), differences across texts are huge with respect to what detail is provided, how it is presented, and what stance is taken toward mathematics and mathematics learning. Even purely mathematical ideas, such as the definition of a fraction, differ considerably. Thus, one might assume differences exist across courses as well.

I have also been unable to find research that analyzes undergraduate mathematics textbooks for teachers with respect to their fit with policy recommendations, current standards for teaching and learning, or current research on learning mathematics. Although many textbooks make the claim that they are “standards-based” or “research-based” it is unclear in some cases whether they incorporate standards or relevant research findings. Such work has been done with respect to K-12 curriculum materials (e.g., the Project 2061 analyses, see Kulm, 1999), but not at the undergraduate level.

There is little research available on the content and pedagogy of undergraduate mathematics courses for preservice K-6 teachers. Although there have been studies of preservice teacher learning of specific topics (see above) and methods courses (Feiman-Nemser & Featherstone, 1992), we know little about what happens in the hundreds of courses taught by mathematics instructors across the country. One can guess that these courses are taught in conventional ways, with textbook problems assigned, new ideas introduced through lecture, and some time given in class for individual practice. But is that hypothesis correct? Is teaching at the college level “by the book” (Sosniak & Perlman, 1990; Stodolsky, 1989) as it often seems to be at the secondary level? If it is by the book, which book(s), and by whom?

Research Design, Methods, and Work Plan

The questions of the proposed research -- What opportunities to learn are provided to prospective elementary teachers in their undergraduate mathematics education, and what do they learn? How do teachers’ opportunities to learn mathematics relate to research and policy on teacher knowledge? – will be investigated in several strands of
work. The research consists of multi-method, multilevel nested case studies, using state, institution, course, instructor, and student as the levels of analysis. Methods will include quantitative (surveys of mathematics departments and instructors, assessments of teacher knowledge), qualitative (interviews with textbook authors, observations and interviews at selected institutions) and analytic (textbook and policy analyses) work. Data sources for the project are both external and internal (Figure 1). The internal data will be generated by the project, while external sources include data sets and documents to be used in analyses and in selection of sites.

*Study of Mathematics Textbooks*

The first part of the study, already under way with support from the College of Education at MSU and the Center for Proficiency in Teaching Mathematics at the University of Michigan, is an analysis of textbooks written specifically for undergraduate mathematics courses for prospective elementary teachers. We have analyzed the 18 books currently available with respect to their overall content and three specific topics: fractions, multiplication, and reasoning and proof (Wallace, Stylianides, & Siedel, 2004). Within each of these topics, we consider not only the mathematical treatment of the topic itself but also the nature of mathematics portrayed, implicit and explicit goals for teacher learning, implied views of teacher knowledge, and the relationship of the topic to the larger body of mathematics and to important ideas in and about mathematics. The rationale for each of the topics is explained briefly below.

*Fractions* is a key topic in elementary education because of the difficulties it presents to K-8 students and teachers and its importance for later mathematics. Students’ procedural and conceptual problems with fractions are well-documented (Barnett et al., 1994; Carpenter et al., 1993; Kamii & Clark, 1995; Mack, 1995). Wu (2001) has argued that correct and thorough understanding of fractions, along with computational fluency, are key to later success in algebra. Tirosi and colleagues’ (1999) study of prospective elementary teachers suggests that their knowledge of fractions is “rigid and segmented,” that they “could not produce adequate representations of rational number concepts or operations with rational numbers, and lacked a representation of mathematics as an organized and structured body of knowledge” (p. 10). Their sample included 147 prospective elementary teachers of whom 26 were mathematics majors. Fractions is included in every textbook we have studied, typically as a focal topic in early chapters.

*Multiplication* is a topic that permeates the K-8 curriculum. Research on learning multiplication indicates that it is conceptually and procedurally easy for students and teachers in some cases (whole numbers, multiplication as repeated addition) and quite difficult in others (multiplication of negative integers) (Greer, 1993; Harel & Confrey, 1994). Multiplication as taught in US schools often nurtures students’ misconceptions that interfere with later learning. The same misconceptions are often held by preservice teachers (Graeber et al., 1989). Conceptual complexities abound in understanding multiplication, including coming to reason quantitatively rather than numerically (Thompson, 1994), and using “intensive quantities” in proportional reasoning (Kaput & West, 1994).
Multiplication is a central topic in every textbook we have analyzed, sometimes as a stand-alone chapter (Beckmann, 2004), but more often as a strand that runs through separate chapters on number systems (e.g., Billstein et al., 2001).

*Reasoning and proof* are key to mathematics as a discipline and to learning mathematics with understanding (Ball & Bass, 2003; CBMS, 2001; Hanna, 1995). Although not topics in the same way that fractions and multiplication are, they are fundamental to doing mathematics and thinking mathematically. The *Principles and Standards of School Mathematics* (NCTM, 2000) recommend that reasoning and proof be included as a central element throughout school mathematics. Research suggests, however, that reasoning and proof cause difficulty for teachers and students at all grade levels (Chazan, 1993; Fischbein & Kedem, 1982; Knuth, 2002; Martin & Harel, 1989; Senk, 1985; Simon & Blume, 1996). Research by Ma (1998) and Simon and Blume (1996) indicates that, in the US, prospective teachers are not inclined to seek mathematical justification, but rather are content to reason by example and to accept faulty explanations. Preliminary analysis indicates that undergraduate mathematics textbooks for teachers take a variety of approaches to reasoning and proof, from making reasoning the centerpiece of the text (Darken, 2003) to including it as an optional appendix (Musser, Burger, & Peterson, 2002). Most books pay some attention to reasoning but leave much of the substance implicit in the way they do or present mathematics.

From the textbook analysis, I am developing constructs that will be used and refined in the author interviews and measured in surveys of mathematics departments and instructors. The initial phase of textbook analysis is complete. We have developed analytic frames for each focal topics, analyzed each book with respect to overall topical coverage and the focal topics, and begun comparisons and interpretations based on these data. In the next phase of analysis, we will correlate textbooks with standards and policies (e.g., CBMS 2001 and state certification requirements).

*Interviews with authors* of each of the textbooks will be conducted to understand their intentions for the book’s use, especially their ideas and beliefs about the mathematical knowledge elementary teachers need. Initial analyses suggest that several of the books include much more than could reasonably be taught in a short sequence of courses (1 – 3 semesters). The authors’ intentions concerning use of their books, what students might learn through their use, and how the books came to their current form will be explored in the interviews. As with textbooks written for K-12 mathematics courses, some of these books could be seen as providing a “mile wide inch deep” approach to the mathematics of elementary school. Others take a specific point of view such as problem solving (Masingila, Lester, & Raymond, 2002), “teaching for understanding” (Center for Research in Mathematics and Science Education, 2000), or presenting mathematics as a rigorous and logical discipline (Jensen, 2003). Based on our analysis of his/her book, I will ask each author specific questions about the focal topics and probe the constructs described below.

*Survey of mathematics departments and instructors*

The goal of this part of the research is to understand what is intended and taught in mathematics courses for K-6 preservice teachers. To accomplish this, I will first select a number of states in which to survey instructors who teach these courses. States are an important unit of analysis to consider when studying teacher preparation, for there is significant variation in state policies that govern teacher certification. I will select states using a number of indicators related to relevant policies. First, using data from NAEP,
SASS, and TIMSS, I will select two states in which students have historically high achievement in K-6 mathematics over the last decade and two states in which mathematics achievement has been low. The states will be selected to meet the following conditions: (a) teachers in the state are certified primarily by in-state institutions; (b) the graduates from in-state certifying institutions tend to get jobs in-state and remain in-state (Loeb & Reininger, 2004); (c) a few large institutions provide a major proportion of the teacher candidates in the state; and (d) at these large institutions, or in the state as a whole, prospective elementary teachers are required to take at least two mathematics courses for certification.

The rationale for these conditions follows from a basic hypothesis of the study, which is that undergraduate mathematics courses for teachers make a difference. The first condition is based on the hypothesis that high or improving achievement may be related to teachers’ mathematical knowledge, which may be related to their undergraduate mathematics education. The second and third provide the possibility that teachers’ mathematical learning is at least in part a product of education that took place within the state. This will provide some basis for comparisons across states. The fourth condition will allow me to concentrate the research in a smaller number of institutions that likely have an impact on teachers in the state. The final condition also provides a basis for comparison across states, and guarantees that institutions will offer courses that include substantial numbers of preservice teachers. States will be matched to take into account population distribution across variables such as socio-economic status.

Once states are selected, I will survey all mathematics departments (via the chair or an assistant chair with responsibility for curriculum and staffing) that certify elementary and middle school teachers to learn what mathematics courses they offer, what they require, how they organize and staff courses, who teaches these courses, and what the prerequisites are. I will also seek information about how they coordinate with mathematics methods instruction and with the certifying department. I will ask whether there are any documents available from recent reviews of the institution, either for the purposes of teacher certification or for program review. I will include all two- and four-year institutions that participate in teacher preparation, since many community colleges are now responsible for teaching the basic mathematics courses for elementary teachers (National Science Foundation, 1999). From these surveys, I will develop a population from which to sample instructors. I will survey a random sample of instructors within each state, over-sampling at the institutions in which I will conduct fieldwork. The collection of program review documents will provide information for looking at important variability across these sites and may provide richer information than a survey would allow.

It is likely that many of the courses elementary teachers are expected to take are not designed specifically for them. Instead, they take courses designed for general, non-technical students. One result from this research will be a better understanding of what kind of courses elementary teachers take, and what those courses include. The CBMS annual survey of mathematics departments (Lutzer, Maxwell, & Rodi, 2002) provides some information about these questions. We know for example that over 40% of mathematics departments that offer mathematics courses for teachers require two courses, with over 70% requiring one to three courses. From CBMS, we also know that there were approximately 68,000 students taking such courses at four-year colleges and 17,000 at two-year colleges in 2000. These data suggest that there is much to study, and many details that are as yet unexplored.
Many aspects of these surveys will be determined with help from my advisory committee, including: piloting the survey; selecting states; developing the population of instructors; and determining the number of instructors to include in the sample. In addition to asking demographic questions about their position, experience, and background, the survey of instructors will measure constructs developed in the textbook analysis, including goals of the course, conception of the course, material use, pedagogy, conceptions of mathematics knowledge for teaching, and assessments of student (prospective teacher) learning. I have developed a preliminary construct map (Figure 2) that shows the relationship between these constructs and the data sources and outcomes. Constructs in the diagram are explained below, after a description of the fieldwork component of the study.

**Fieldwork**

Within the high achieving states, I will identify two institutions per state to be the subject of detailed case studies, matching institutions across the states with respect to population variables. I will restrict the case studies to high achieving states since this part of the work will be limited in scope and, consistent with theories of case study research, use similar rather than divergent cases allowing for a “replication logic” (Yin, 1993, p. 34). Using data from the CBMS survey and from my own survey of departments, I will choose institutions that are representative in terms of the number of courses required, the number of graduates, credentials of instructors, and other characteristics. At the selected institutions, I will conduct case studies of three sections of the mathematics courses for elementary teachers, including interviews with the instructors, students (prospective teachers), and other members of the mathematics faculty. These protocols will investigate opportunities to learn and teacher learning, and will provide narrative and descriptive data about the courses.

General faculty interviews will explore the departmental participation in teacher education, including how decisions about courses and coordination are made. Interviews with instructors will probe in greater depth than allowed on the survey, with direct questions about opportunities to learn and assessment of student learning. The observation protocol will include observing in each selected section for three hours (one week’s classes), with attention to how time is spent, how the textbook or other materials are used, how the instructor and students interact, and what subject matter is addressed. These observations will provide for triangulation with survey data.

An important part of the fieldwork will be assessing learning in mathematics courses at the case study institutions. I will use items and methods from Hill and colleagues (Hill, Rowan et al., 2004; Hill, Schilling, & Ball, 2004); NCRTL (Kennedy, Ball, & McDiarmid, 1993); Ma (1998); TIMSS (1999) to develop an assessment tool. I will use this tool in the case study classes and possibly in a wider selection of classes in the case study institution. My plan is to link assessment data with instructor data. That is, I will assess students who are in the classes of instructors selected for the survey. I will administer pre- and post-surveys to these students and will be in a position to do follow-up work with them in future research.

**Constructs**

The constructs identified in Figure 2 will be investigated across protocols and research sites. Preliminary definitions of these constructs and data sources for their investigation are discussed here:

- **Goals** include the intentions of the textbook, author, or instructor. For the textbook analysis we have identified several goals that will be used in developing the surveys. These
are: knowing the mathematics of elementary school; understanding mathematics in depth; being able to solve problems; understanding mathematics as a discipline; developing a rigorous foundation for mathematics teaching and learning; and improving attitudes toward mathematics. While these goals are not mutually exclusive, books tend to emphasize one (or some) more than others, and we expect instructors do the same. Interviews with authors will help solidify these categories as we probe what they intend in their development of the texts. Once categories are clear, we will write survey items to investigate the goals mathematics instructors have for their courses.

Conceptions of the course, while related to goals, looks at somewhat different aspects of course planning and implementation. This includes giving students opportunities to do mathematics; covering a few topics in depth; covering a significant part of the topics in the elementary curriculum; giving students opportunities to solve complex problems; and helping students learn to learn mathematics. This list will be fleshed out in the author interviews and survey piloting.

Mathematics content and material use will be covered on the surveys of mathematics departments and instructors. We want to learn whether textbooks or other materials are used, what specific materials are used, and what topics are covered in the course. The textbook analysis has provided us with a list of topics that we will use in the survey, relating it either to specific texts in the case where a textbook is used or to the general coverage of the courses in other cases.

Pedagogy will be included in the instructor survey, aiming to find out modes of teaching. Lecture, discussion, problem solving sessions, homework review, and other categories will be explored, as to their use and frequency. Previous work on K-12 teaching has made use of categorizations of classroom activity, and we will draw on these studies (e.g., Porter and colleagues work on content determinants, Porter, Floden, Freeman, Schmidt, & Schwille, 1986). Pedagogy will also be explored in field observations.

Mathematical knowledge for teaching will also be included in the instructor survey, with questions aiming to understand what the instructors think it is important for teachers to know and whether this is different than what non-teachers might learn. This construct will be
explored in interviews with textbook authors, textbook analyses, instructor survey and student assessments.

Assessment of learning will be included in the instructor survey, with questions about how they assess student learning, and whether they are satisfied with what students learn. Learning will be directly assessed in the instrument administered to students in the undergraduate courses in pre- and post-tests.

Opportunities to learn (OTL) will be measured in the instructor survey through questions that ascertain what materials they use, what topics they cover, and how much time they spend on focal topics. OTL data will also be derived from textbooks and their authors; from surveys of mathematics departments and their instructors; and from field observations. OTL is a construct that has a two-decade history in educational research (Floden, 2002). At the most basic level, it was defined as whether students have had explicit chances to learn a given topic in a given educational context. This definition has been elaborated and expanded in evermore complex instantiations in national and international assessments of students’ learning, in efforts to understand differences across countries or even states or districts (McDonnell, 1995). Floden points out that OTL can be assessed at multiple levels, from state to district to school to classroom, with each one getting closer to what students actually see and do. For the proposed research, I will assess OTL at multiple levels: state and institutional requirements, textbooks, instructors, and classes.

Data Analysis

Textbooks analysis provides data to develop constructs for the other parts of the study. The constructs for focal mathematical topics will include definitions, representations, sequence, and emphasis, the latter two based on the work of Schmidt (Schmidt, McKnight, Valverde et al., 1997) and Porter (Porter et al., 1986), respectively. Each textbook will have values on each of these dimensions, along with others to be defined. For each specific item within a topic, I will have a two part code: a binary code for whether the OTL exists and a variable for its relative strength and emphasis. For example, if a textbook includes a representation of fractions as division, the opportunity to learn that representation has a value of 1 for that textbook. The other value will be determined based on how much space it occupies and how it is presented in the text. The analysis will also be used to create a scale of correspondence with the CBMS (2001) recommendations, NCTM (2000) Standards, and state requirements.

Author interviews will be coded with respect to the constructs and then matched with coding for the book. For example, if the author says that he has provided a rigorous definition for “fraction” but I have coded it as an intuitive definition, I will reevaluate to understand the difference. I will ask the authors about their sense of the emphasis of particular items in the focal topics to confirm the prior analysis. I will use the results of this analysis in several ways: to describe and understand the books fully; to interpret instructor responses to survey questions with respect to the book they use; and to aid in developing an observation protocol for the fieldwork, and as a proxy measure of OTL in courses that use the book. In addition, I expect author interviews to provide important data about sales, who is using their books, and new ideas for what would be interesting to learn about courses for prospective elementary teachers.

Mathematics department surveys will provide descriptive data for the four states in the study, including how many courses are offered for elementary preservice teachers; what courses they take in addition to those required; who teaches the courses (by rank, experience,
age, sex); how many sections and students they teach each year. This survey will also include items that will be analyzed using Item Response Theory with respect to the constructs, and used in comparison to other mathematics departments, and to surveys of instructors within the department. I will use Hierarchical Linear Modeling techniques to make comparisons across institutions and states.

**Instructor surveys** will provide descriptive data about who teaches these courses, along with data for evaluating the constructs related to general attributes of the course and their teaching, and the focal mathematical topics. The survey will include some very specific questions about a small subset of the focal areas. For example, within multiplication, the survey will focus on multiplication of integers. I will use the same techniques for analyzing these surveys – IRT to assess instructor’s positions with respect to the constructs, HLM to make comparisons across classes, institutions, and states.

**Observations of classes** will be used to validate instructor responses (on a small scale) and as illustrative data to flesh out the meaning of the surveys. I will observe classes when they are working on one of the specific areas of interest within a focal topic whenever possible.

**Teacher pre- and post- assessments** will include some general questions along with specific questions about the focal topics (e.g., multiplication of integers). These will be used to understand what teachers have learned, using IRT techniques to analyze measures of particular aspects of teacher knowledge. Teacher learning will be correlated with instructor and departmental data to look for significant relationships between the constructs of interest and student learning. These will be compared across institutions and states as well.

**Work Plan**

There are four streams of work that constitute this research: (a) Mathematics textbook analysis and interviews with authors; (b) Mathematics department and instructor surveys; (c) Field observations and interviews; (d) Assessments of teacher learning. I have developed a five-year plan for this work as shown in Figure 3. To accomplish the proposed work, I will have two half-time graduate assistants one of whom will help me with surveys and quantitative analysis, and the other with field work design and implementation.

I will need expertise and advice in several areas, and have created an advisory committee for this purpose. The board includes Drs. Robert Floden and Suzanne Wilson (education) and Dr. Joan Ferrini-Mundy (education and mathematics) from Michigan State; Drs. Deborah Ball and Ed Silver (education) and Hyman Bass (mathematics) from the University of Michigan; Dr. Sybilla Beckmann (mathematics; also a textbook author) from the University of Georgia; Dr. Roger Howe (mathematics) from Yale University; Dr. Zalman Usiskin (mathematics education) from the University of Chicago. I will add one more mathematician to the team. In addition, Dr. Mark Reckase from MSU’s Measurement and Quantitative Methods program has agreed to consult with me on statistical issues. I will meet with the advisory board three times during the project, in years 1, 3 & 4, and I will call on individuals as needed to help me with specific problems. Michigan State University is the site of many projects with which my work intersects, including the Center for the Study of Mathematics Curriculum; the Teachers for a New Era project funded by the Carnegie Corporation, which is in the process of redesigning undergraduate education for prospective teachers; the Michigan/Ohio Math/Science partnership PROM/SE; the international Teacher Education Study in Mathematics, TEDS-M; and the ROLE Knowledge of Algebra for Teaching (KAT) project, on which I am a co-PI. I plan to coordinate with these projects to
benefit from their work, and to share mine. Drs. Floden, Ferrini-Mundy, and Usiskin are each involved in one or more of these projects, and I am a co-PI on the KAT project.

Dissemination

As indicated in the project flow chart (Figure 3), my plan is to produce research reports at various stages of the work, beginning immediately with the textbook analyses. There has already been considerable interest in the textbook work, with proposals accepted at the Joint meeting of the Mathematical Association of America and the American Mathematical Society, 2004, and NCTM (National Council of Teachers of Mathematics) 2004. I will soon submit a paper on textbook analysis to the Journal for Research in Mathematics Education (JRME). As the work progresses, I plan to submit articles to The American Mathematical Monthly (writing on mathematical knowledge of prospective elementary teachers), Research in Collegiate Mathematics Education (what is taught in undergraduate mathematics courses for elementary teachers), Journal of Mathematics Teacher Education (textbook analysis), and Educational Evaluation, and Policy Analysis (OTL and implications for policy).

Educational Plan

The objectives of my educational plan are to dual: (a) to contribute to the improvement of the mathematical education of prospective elementary teachers at MSU and, more ambitiously, nationwide; and (b) to improve my professional skills in both teaching and research. During the five years of this research, I plan to teach both mathematics and methods courses. Two different versions of the mathematics courses are taught at MSU, one using Parker & Baldridge (2003) and the other using materials developed at San Diego State (Center for Research in Mathematics and Science Education, 2000). I will gain insights into the impact of different materials on student learning and will try different assessments not practical on a large scale. As instructor, I can interrogate my own mathematical knowledge, a different level of mathematical knowledge for teaching and a logical next step given the research I propose above.
I will also redesign a PhD seminar I teach, “Teaching and Technology.” In this course, I conceptualize technology broadly as any tool for teaching, including conventional technologies such as curriculum materials. I want to shift the focus from practice to teacher learning and knowledge. One part of my previous research has been about teaching and learning to teach with technology, an interest that has always been more about teaching than about technology. Technology has provided an interesting site for my research because it reveals aspects of teaching that are often obscured by routines and expertise. I see the research proposed here as an extension of my work on teaching and learning to teach, and, in particular, on using resources (technologies) in teaching. The PhD course will provide a venue for new thinking and reading about teacher knowledge.

Finally, with the support of my department, I plan to enroll in an intensive summer program to learn more about survey development and analysis and other statistical techniques. Although my background is in mathematics and I taught elementary statistics as a graduate student, my knowledge of survey methods and more advanced statistical methods needs improvement to do the kind of work I want to do.

As a result of this educational work, I will be in a good position both to participate in and to research the mathematical education of prospective K-6 teachers and the problem of teacher knowledge. The research and educational plan proposed here opens up a number of very interesting possibilities for ongoing research, discussed in the next section.

Future Research

It would make sense to develop the textbook analysis into an ongoing repository for information and data about mathematics textbooks for K-6 preservice courses. This could be a database, updated regularly, that includes data about the content of the books and information about where books are being used. We have learned that the market changes rapidly, with books going out of print, new editions being published, and new books coming onto the market every year. A second kind of work related to the textbook studies is historical analysis of how these books have changed over time. A third type of work is to expand this analysis to include methods books.

In the instructor/course strand, future research could include an intervention study, comparing results for carefully designed interventions. The survey could be extended to additional states. Since we know so little about what is taught in these courses, data from a larger population of instructors, courses and states would be interesting and potentially useful for policy and practice. Another interesting possibility is studying instructors’ mathematical knowledge.

In the case study work, an assessment tool for preservice teacher’s mathematical knowledge is valuable as a guide to what we should teach and as feedback about the success of our teaching. Other projects at MSU (PROM/SE, TEDS-M, TNE, and KAT) are also working to develop assessments of teachers’ mathematical knowledge. I will work with them and share items and results as each of these projects progresses.

The most important future research, however, will be following students in the initial study into their induction years to look at the impact of their mathematical education on their practice, which, if the work proposed here is successful, I plan to do.

Merit Criteria

The intellectual merit of this research flows from the critical need for better mathematics teachers at all levels of education. This research will contribute to understanding whether prospective elementary teachers are exposed to content that they need
for successful teaching, and whether they are learning that content. Given the urgency and exposure of the problem of teacher knowledge, it is surprising to me that research on this subject is so rare. From the survey data, we will learn what instructors are teaching, informing policy makers and practitioners as they consider how to improve the undergraduate education of prospective teachers. We will have data about teacher learning, possibly providing insights into which models are successful in helping teachers learn relevant mathematics.

We will have data about the fit between undergraduate mathematics courses and emerging conceptions of mathematical knowledge for teaching. The research will contribute to theories of teacher knowledge by investigating the points of view of textbook authors who have thought long and hard about the mathematics they include in their books and how they present it, and instructors who deal daily with prospective teachers’ learning. This is new research in areas about which little is known, but which could have important implications for teacher education.

I have identified four broader impacts of this work. (a) We may learn about particular courses and textbooks that make a difference in teachers’ knowledge, and in that case, will have data that provides insights into why and how these successes occur. This research could result in a more complete understanding of the mathematics teachers need and could influence instructors of mathematics courses for elementary teachers to teach that mathematics directly. Based on informal discussions with two of the authors, I know that their purposes in writing their books were similarly ambitious (Beckmann, 2004; Parker, 2003). (b) The study will produce data about textbooks, courses, and instructor characteristics that may be useful at a practical level to institutions, departments, and instructors as they make critical decisions about what to teach and how to teach it. (c) The textbook analysis framework could also become a model with wider application, for it is content- (rather than market-) driven, and provides a conceptual basis for considering the qualities of textbooks. (d) Finally, the instruments -- surveys of instructors and departments, and assessments of preservice teachers -- could be more widely used for a variety of purposes. Hyman Bass (personal conversation) urged me to administer the instructor survey to a random national sample, a scope which seems out of reach for the research proposed here, but could certainly be a goal for the future.

As the Principal Investigator for this work, I am in a good position to do this research because of my background as a mathematician and mathematics teacher; my ongoing participation in mathematics teacher education; my interest in teacher learning and teacher knowledge; and my research on use of materials in teaching. I have been continuously involved in research on mathematics teacher learning and teacher knowledge since 1995, including working with some of the leaders in the field at both the MSU and the University of Michigan. I am currently working on another project about teacher knowledge, the NSF ROLE KAT project, which is investigating algebra knowledge for teaching. Along with Ferrini-Mundy, Floden, and Senk, I am currently a co-PI on the Kat project, which started on June 15, 2004. In prior experience with NSF, I contributed to the first ROLE Algebra project (PI’s Chazan, Ferrini-Mundy, Floden, and Senk), doing analysis of video and developing an interview protocol for practicing algebra teachers. As a graduate student at the University of Michigan, I worked on the NASA, NSF, DARPA Digital Libraries Initiative project and published an article about our early work on using the Internet as a classroom resource (Wallace, Kupperman, Krajcik, & Soloway, 2000).
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