Context of the Study

• Agreement on the overall inadequacy of elementary and middle school teachers’ mathematical knowledge
• Need a better understanding of the mathematics elementary teachers need to know and how they can learn it
• Current research, national policy documents, and professional standards offer new views of mathematical knowledge for teaching
Questions

What mathematics are we teaching prospective elementary and middle school teachers in undergraduate courses?

What mathematics are they learning?

Who teaches this mathematics: Education, Mathematics, or both?
Question

What do prospective elementary and middle school teachers have an opportunity to learn in their undergraduate mathematics education?
Plan for the Presentation

• Overview of the study
  – Overall scope
  – Textbook analysis, methods and examples
• Analysis: Multiplication of Integers
• Analysis: Reasoning and Proof
• Conclusions
  – Textbook analysis results
  – Mathematical knowledge for teaching
Parts of the Study

- Identify and analyze **mathematics** textbooks
- Interview textbook authors
- Analyze state requirements, state and national policies, professional standards
- Survey instructors
- Interview instructors
- Review international textbooks and requirements
- Investigate the history of such textbooks
- Identify and catalog **methods** textbooks
Analysis of Textbooks

• Undergraduate mathematics textbooks
• For prospective elementary and middle school teachers
• For courses usually taught in mathematics departments
• For one, two, three, or more semesters, depending on state requirements
Analysis of Textbooks: Texts

- 18 Books identified
  - 2 self-published
  - 2 preliminary editions
- These are all such books currently in print

We are currently polling publishers to be sure we have found them all.

One we found late -- some slides refer to 17 books


Jensen, Gary R. 2003. *Fundamentals of Arithmetic*


http://www.msu.edu/~ravenmw/


Masingila, Joanna O., Frank K. Lester, and Anne M. Raymond. 2002. *Mathematics for elementary teachers via problem solving*


# Textbooks

<table>
<thead>
<tr>
<th>Authors</th>
<th>Number of Chapters</th>
<th>Average Chapter Length</th>
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<tbody>
<tr>
<td>Bassarear (2001)</td>
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<td>57 (26-87)</td>
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<td>Masingila et al. (2002)</td>
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<td>O’Daffer et al. (2002)</td>
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<td>25 (16-37)</td>
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<td>37 (28-64)</td>
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<td>Sonnabend (2004)</td>
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<td>60 (13-95)</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>13.6</strong></td>
<td><strong>49.6 (26.6-79.8)</strong></td>
</tr>
</tbody>
</table>

*For one semester only
Analysis

• Is the content of these textbooks obvious?
• Do they all have the same content?
• How does the content vary?
## PRELIMINARY Data -- subject to change

| TEXT | # | Number of Chapters | Average Chapter Length | Problem Solving | Sets | Reasoning and Proof | Logic | Addition | Subtraction | Multiplication | Division | Number Systems | Prime Numbers | Fractions | Decimals | Ratios | Exponents | Number Theory | Data and Statistics | Probability | Geometry | Measurement | Transformation (Geometry) | Functions | Algebraic Thinking/Early Algebra | Mental Math | Estimation | Ratio and Proportion |
|------|---|---------------------|------------------------|-------------------|-----|--------------------|-------|----------|------------|----------------|----------|----------------|--------------|----------|----------|--------|---------|----------------|-----------------------|------------------------|-----------|---------|-------------|------------------------|-----------|--------------------------|-----------|-----------|---------------------|
| Bassarear (2001) | 1 | 0 | 59 (38-98) | T | 1 | T | 2 | T | I | I | 3 | T | 5 | 5 | I | 2 | T | 5 | T | 5 | T | 5 | T | 6 | 4 | 7 | 8 | T | 9 | T | 2 | T | 3 | T | 3 | 5 | 6 |
| Beckmann (2003, 2 volumes) | 1 | 2 | 66 (16-124) | I | 1 | I | 2 | I | I | 3 | 4 | 5 | T | 2 | T | 2 | T | 2 | 3 | 4 | 5 | 5 | T | 2 | T | 2 | 2 | 3 | 6 | I | 1 | I | 2 | 7 | 9 | 8 | I | 0 | I | 0 | T | 4 | T | 3 | I | T | 5 |
| Bennett & Nelson (2004) | 1 | 1 | 72 (40-95) | I | 1 | 2 | 2 | I | I | T | 3 | 5 | 6 | T | 3 | 3 | 5 | 5 | 6 | 6 | 6 | 4 | 7 | 8 | 9 | 8 | I | 0 | I | 0 | T | 4 | T | 3 | I | T | 5 |
| Billstein et al. (2004) | 1 | 2 | 69 (53-87) | I | 1 | 2 | T | I | T | 3 | 4 | 5 | 6 | 3 | 2 | 5 | 4 | 6 | 6 | 6 | 4 | 8 | 7 | 9 | I | 0 | I | 2 | 2 | I | T | 1 | T | 3 | 3 | 5 | 6 | T | 5 |
| CRMSE (2000-2001, 2 volumes) | 1 | 9 | 20 (8-81) | I | 1 | A | T | 2, 3, 7, 8 | AT | 4 | 6 | A | T | 8 | A | T | 4 | 6 | 6 | 6 | 4 | 8 | 7 | 9 | I | 0 | I | 2 | 2 | I | T | 1 | T | 3 | 3 | 5 | 6 | T | 5 |
| Darken (2003) | 1 | 2 | 57 (32-98) | I | 2 | T | 2 | I | I | T | 1 | 4 | 5 | 6 | 3 | T | 1 | 6 | T | 1 | I | 6 | T | 4 | 6 | T | 4 | 3 | 6 | 2 | 7 | 8 | 9 | I | 0 | I | I | T | 3 | 1 | 2 | T | 4 | T | 4 | T | 1 | 5 |

### KEY: # indicates chapter number for topics that correspond to chapters, where the topic is part of the chapter title.

T# indicates that the topic is included as a section in chapters (#).

A indicates topic is in appendix or supplementary pages.

# means topic appears in the index # times but is not a chapter title or section heading.
Analysis of Textbooks: Focus

- Multiplication
- Reasoning and Proof
- Fractions (Rational Numbers)
**Multiplication**

- A basic topic in elementary mathematics
- Cuts across grade levels
- Cuts across number systems
  - From whole numbers to fractions and decimals to algebraic expressions
- Conceptually easy in some cases, conceptually difficult in others

Repeated addition for whole numbers

Multiplication of two negative integers
Reasoning and Proof

- A mathematical way of thinking and doing
- Emphasized in national standards
- Students and teachers have difficulty with reasoning and proof
Fractions

• Numbers
• Taught later in the elementary curriculum
• A key concept for learning algebra
• Conceptually and procedurally difficult for many teachers and many students
Analysis of Textbooks: Method

• Review of research on each topic:
  – How students learn
  – Particular trouble spots for teaching and learning

• Develop a list of topics, concepts, and procedures for each focal area
  – Content
  – Topic development
  – Trouble spots

• “Code” and comment on each book with respect to the list
Methods: Fractions

• Primary Categories:
  – Definition
  – Sequence
  – Coverage
  – Representations and models
  – Properties
  – Word problems, examples, & applications
  – Pedagogy
Fractions

• Definition (For each book, note which definition is primary, if any)
  – Number line definition
  – Set theoretic definition (ordered pair)
  – Definition only by example -- intuitive definition
  – As a number system -- rational numbers
  – As an operation -- division
  – Other
## Fractions Example 1


<table>
<thead>
<tr>
<th>Set theoretic definition (ordered pair)</th>
<th>N</th>
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<tr>
<td>Definition only by example -- intuitive definition</td>
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<tr>
<td>As a number system -- rational numbers</td>
<td>Y</td>
</tr>
<tr>
<td>As an operation -- division</td>
<td>Y</td>
</tr>
</tbody>
</table>

As a set of numbers that **includes** the rational numbers:

**Rational numbers:** “A set of numbers of the form $a/b$, where $b \neq 0$ and $a$ and $b$ are integers. Moreover, numbers of the form $a/b$ are solutions to equations of the form $bx = a$. This set, denoted by $Q$, is the set of rational numbers and is defined as follows:

$$Q = \{a/b \mid a \text{ and } b \text{ are integers and } b \neq 0\}$$

(p. 246)

**Relation between rational numbers and fractions:** “$Q$ is a subset of another set of numbers called fractions. Fractions are of the form $a/b$ where $b \neq 0$ but $a$ and $b$ are not necessarily integers. For example, $1/\sqrt{2}$ is a fraction but not a rational number. (In this text we restrict ourselves to fractions where $a$ and $b$ are real numbers, but that restriction is not necessary.)”

(p. 246)

“The rational number $a/b$ [with $a$ on top of $b$] may also be represented as $a/b$ or as $a\div b$. The word fraction is derived from the Latin word fractus meaning ‘to break.’”

(p. 246)
Fractions Example 2

<table>
<thead>
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<tr>
<td>Set theoretic definition (ordered pair)</td>
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</tr>
<tr>
<td>“A (common) fraction is ultimately an ordered pair of whole numbers whose second component is nonzero. For reasons that will be made clear shortly, the symbol 3/4 is more appropriate than the conventional ordered-pair symbol (3, 4).&quot; (p. 334)</td>
<td></td>
</tr>
<tr>
<td>Definition only by example -- intuitive definition</td>
<td>N</td>
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<tr>
<td>As a number system -- rational numbers</td>
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<td>As an operation -- division</td>
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</table>
Plan for the Presentation

• Overview of the study
  – Overall scope
  – Textbook analysis, methods and examples

• Analysis: Multiplication of Integers

• Analysis: Reasoning and Proof

• Conclusions
  – Textbook analysis results
  – Mathematical knowledge for teaching
Multiplication of Integers in Mathematics Textbooks for Prospective Elementary and Middle School Teachers

Helen Siedel
University of Michigan
Reasons for Investigation of This Topic

- Multiplication is a focus topic for our study
- We observed substantial variation in the presentation of multiplication of integers
- The use of real-life contexts and concrete models is awkward
- Negative numbers are challenging for teachers to teach and for children to learn
- More teachers may be teaching about integers
The Challenge

“Even though models for negative numbers may be less intuitive to children than models for fractions and decimals ... children generally find learning about the system of integers to be easier than working with the positive rational numbers. The notation for negative numbers is less complex than that for rational numbers, ....Furthermore, the rules for operating on integers are easier to learn and apply than the corresponding algorithms with fractions. The challenge for teachers is to assist children in understanding why as well as how these rules work.”

Cathcart, Pothier, Vance, & Bzuk, N. A. (2003), *Learning Mathematics in Elementary and Middle Schools*, p. 381
Reasons for Investigation of This Topic

• Multiplication is a focus topic for our study
• We observed substantial variation in the presentation of multiplication of integers
• The use of real-life contexts and concrete models is awkward
• Negative numbers are challenging for teachers to teach and for children to learn
• More teachers may be teaching about integers
Research Question

What are the variables in authors’ presentations?
Method

• List the yes/no variables
• Select those variables that could be considered content variables
• Group the selected variables
• Develop a numerical summary of the yes/no variables
• Analyze the numerical summary
• Identify more complex variables
• Identify topics for further study
## Definition

<table>
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<th>Textbook author(s)</th>
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<td>Musser, Burger, Peterson (2003)</td>
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<td>Parker &amp; Baldridge (2003)</td>
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<td>Sgroi &amp; Sgroi (1993)</td>
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<tr>
<td>Sonnabend (2004)</td>
<td></td>
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</tbody>
</table>
Krause Definition

Definition of Multiplication (•) of Integers
For all whole numbers $m$ and $n$:

\[ m \cdot n = n \cdot m \]

\[ -m \cdot n = n \cdot -m = -(m \cdot n) \]

\[ -m \cdot -n = m \cdot n \]

Krause, 1991, p. 306
Long and DeTemple Theorem

**Theorem** *The Rule of Signs*

Let $m$ and $n$ be positive integers so that $-m$ and $-n$ are negative integers. Then the following are true:

\[
m \cdot (-n) = -mn
\]

\[
(-m) \cdot n = -mn
\]

\[
(-m) \cdot (-n) = mn
\]

Long & DeTemple, 2003, p., 317
### Table: Models

<table>
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<tr>
<th>Textbook author(s)</th>
<th>Patterns</th>
<th>Number line-position</th>
<th>Number line-changing direction</th>
<th>Repeated addition</th>
<th>Sets of objects</th>
<th>Charged field or signed chips</th>
<th>Debit/Credit</th>
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<th>Time (ago)</th>
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<td><strong>4</strong></td>
<td><strong>4</strong></td>
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</table>
Patterns
From Masingila, p. 90

**DID YOU KNOW**

The rules for multiplying negative numbers by negative numbers makes sense when you analyze the pattern of product found by multiplying a given integer by a series of positive and negative integers. Observe:

\[
\begin{align*}
4 \times -3 & = -12 \\
3 \times -3 & = -9 \\
2 \times -3 & = -6 \\
1 \times -3 & = -3 \\
0 \times -3 & = 0 \\
-1 \times -3 & = 3 \\
-2 \times -3 & = 6 \\
-3 \times -3 & = 9 \\
\text{etc...}
\end{align*}
\]

As the first factor decreases by 1, the product of the first factor and \(-3\) increases by 3. Thus, the pattern suggests that a negative times a negative should yield a positive number to continue the pattern.
Number Line Model
From O’Daffer, p. 258

FIGURE 5.14 | Another number line model for integer multiplication.
Chips Model
From O’Daffer et al., p. 255

**Action 1: Putting 2 Counters in the Bag (2)**

Positive × Positive
Do action 1 three times.
3(2) = ?

Does the bag have more (+) or less (−) counters than before?
How many more or less?

Negative × Positive
Do the opposite of action 1 three times.
−3(2) = ?

Does the bag have more (+) or less (−) counters than before?
How many more or less?

**Action 2: Taking 2 Counters out of the Bag (−2)**

Positive × Negative
Do action 2 three times.
3(−2) = ?

Does the bag have more (+) or less (−) counters than before?
How many more or less?

Negative × Negative
Do the opposite of action 2 three times.
−3(−2) = ?

Does the bag have more (+) or less (−) counters than before?
How many more or less?

**FIGURE 5.11** Using counters to model integer multiplication.
Models Summary

• Repeated addition: 13
• Patterns: 9
• Sets of objects: 7
• Number Line: 6
• Debit/Credit: 6
### Table: Notation $-m$

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<tr>
<th>Textbook author(s)</th>
<th>Raised negative</th>
<th>Raised positive</th>
<th>Discussion</th>
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<tr>
<td>Troutman &amp; Lichtenberg</td>
<td>Y</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>7</strong></td>
<td><strong>3</strong></td>
<td><strong>7</strong></td>
</tr>
</tbody>
</table>
Notation for Negative Integers

Ultimately, however, these texts invariably change to the standard notation …. We feel that it causes less confusion to do this at the outset, stressing that the context makes it clear when ‘subtract’ is meant as opposed to ‘the negative of.’

Long and DeTemple, 2003, p. 285
Properties

- Lists Properties: 14 books
- Uses Properties: 14 books
Other ideas

- Opposite: 17 books
- Additive Inverse: 11 books
- Absolute Value: 13 books
Findings

• Definition
• Vocabulary
• Variable use of properties
Finding #1: Few authors specify a definition for the multiplication of integers
Finding #2: Assumptions are made about the vocabulary of prospective teachers
Finding #3: These books do not suggest consensus about how to make sense of multiplication of integers
Reasoning and Proof in Mathematics Textbooks for Prospective Elementary and Middle School Teachers

Andreas Stylianides
University of Michigan
Plan for the Presentation

• Overview of the study
  – Overall scope
  – Textbook analysis, methods and examples
• Analysis: Multiplication of Integers
• Analysis: Reasoning and Proof
• Conclusions
  – Textbook analysis results
  – Mathematics for teaching
Types of books

- Multiple Editions?
- Role of Publishers?
- Changing Standards?
- Changing Expectations?
Big Ideas?

- Hard to find the big mathematical ideas in some of the texts
- Hard to find connections across topics
- Some of the texts might allow one to view mathematics as a bundle of loosely related topics and rules
Conceptions of knowledge

Methods

Mathematics

Mathematics for Teaching
Conceptions of Knowledge

A common error among children when first learning multiplication is the following:

\[
\begin{array}{c}
35 \\
x 25 \\
1025 \\
615 \\
1640 \\
\end{array}
\]

Methods

Mathematics

for Teaching

Algorithms for multidigit multiplication can be proved using the distributive property.

This example is from Ball, 2001

\[
\begin{array}{c|c|c}
A & B & C \\
\hline
35 & 35 & 35 \\
\times 25 & \times 25 & \times 25 \\
125 & 175 & 25 \\
+ 75 & + 700 & + 150 \\
875 & 875 & 100 \\
& & + 600 \\
& & 875 \\
\end{array}
\]
Conceptions of Knowledge

Methods

There are several models for multiplication of negative integers that teachers need to know: signed chips, number line, temperature, debit/credit.

Mathematics

The axioms for the field of real numbers can be used to derive the rules for operations with negative integers, and they imply that these rules must be what they are.

Mathematics for Teaching

Each model has particular mathematical characteristics, including both benefits and drawbacks. Teachers need to know what each entails and what is given up when using each model.
Conceptions of Mathematical Knowledge for Teaching Mathematics Methods Mathematics for Teaching

What is the knowledge here?

What is in this gap?

Methods Mathematics

Mathematics for Teaching

Methods Mathematics

Mathematics for Teaching
Conceptions of Knowledge

Who teaches this?

Methods

Mathematics

Mathematics for Teaching

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DRAFT: Do not cite or quote
Conclusions

• Next stage of analysis
  – Approach
  – Goals
  – Assessment
  – Conceptions of knowledge

What kind of teachers do we want?

• Problem solving
• Mathematical thinking
• Fewer topics, in depth
• Logical development
Conclusions

• What mathematics is OFFERED to prospective elementary and middle school teachers?
  – Lots of variation across books
  – Many possibilities for constructing a course
  – Often hard to tell what the textbook authors consider critical

• Across the texts, the line between “method” and “mathematics” is not clearly drawn

• The next stages of the research will be telling
  – What do authors intend?
  – How do instructors use the books?