Examples of How Semiotics Can be Used to Teach Fractions to Prospective Elementary School Teachers

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Purpose of this Study

In this paper we consider how semiotics instruction can be used to help preservice elementary school teachers learn about fractions. We describe how one professor instructed preservice elementary school teachers using a dyadic, or two-part, semiotic framework developed by Ferdinand de Saussure (1957). By using this framework, the teacher was able both to decompose the problems and to give the preservice teachers a way to understand how their students may view the problems. We then show how another teacher who uses pattern blocks to instruct students about fractions could also use semiotics as a framework.

Theoretical Framework: Fractions as a Semiotic issue

Semiotics, broadly conceived, "is concerned with everything that can be taken as a sign" (Eco, 1976, p. 7), including "images, gestures, musical sounds, objects…these constitute, if not *languages*, at least systems of signification" (Barthes, 1967, p. 9). De Saussure (1957) described a linguistic sign as a two-sided entity made up of a signifier and signified. A signifier is the material aspect of the sign whereas the signified is the mental concept associated with the material symbol. For example, the English word "tree" is made up of the material sounds /t/, /r/, and /e/ (the signifier) as well as the mental concept we each hold of what it means to be a "tree" (the signified).

The meaning of a sign, however, is not contained within the signifier or signified alone, but develops within a phrase and within a community. To make sense of the sign, we need to take not only the signifer and signified into account, but also the context that contains the sign. One type of sign may be signified by a written symbol, such as a fraction (a/b). We are interested in how to use semiotics to help preservice elementary school teachers move beyond the use of fraction symbols to a conceptual and transferable understanding of what those symbols mean.

Taking the idea of fractions as signs, we can see how it can be difficult for students to make sense of them. There are many different definitions of fractions, but for the sake of this paper, we use Lamon's (2007) definition as "non-negative rational numbers" written in the a/b notation (p. 635). Preservice elementary teachers often have trouble understanding and ultimately teaching future students about fractions (Graeber, Tirosh, & Glover, 1989; Harel et al., 1994; Simon & Blume, 1994). One key issue that prospective teachers face is how a fraction is related to division, is a type of multiplication, or is a ratio of some sort (Ni, 2001).

A single signifier of a fraction can take on multiple meanings. Consider the following uses of rational numbers[i] taken from a popular book used to teacher prospective elementary teachers mathematics (Billstein, Libeskind, Lott, 2007, p. 299):

Table 5-1 Uses of Rational Numbers

Use	Example
Division problem or solution to a multiplication problem	The solution to 2x = 3 is 3/2
Partition, or part, of a whole	Joe received ½ of Mary's salary each month for alimony.
Ratio	The ratio of Republicans to Democrats in the Senate is three to five
Probability	When you toss a fair coin, the probability of getting heads is ½.

In the example, a fraction could be used as division, to partition something, as a ratio, or as a probability. Similarly, Kieren's analysis identifies five "subconstructs": part–whole, ratio, quotient, operator, and measure (Behr et al., 1992; Kieren, 1976). Depending on the context, the meaning of a number – even if it is represented by the same symbol – changes.

We posit that one difficulty students have when trying to understand and teach fractions is that they must navigate across multiple concepts, or signifieds, for the same symbol, or signfier, a relatively common task in spoken and written language, but not necessarily common in the understanding of numbers. We use the phrase 'semiotic dissonance' to describe the difficulty or inability for a person to meaningfully construct a sign from a signified (meaning) and its associated signifier (symbol). Further, when a teacher develops the ability to navigate across meanings, he or she must be able to step back and understand how future students must then navigate across meanings. This understanding is the basic semiotic framework that we will refer to for the rest of this paper.

Method and Data Sources: Fractions in Math Classes for Preservice Elementary Teachers

As part of a larger study that explores the mathematics taught to undergraduate prospective elementary teachers, this paper focuses on two of seven mathematics instructors who were videotaped while they taught fraction lessons. These two instructors were selected because of the unique approaches they took to teaching fractions. The video data was supplemented by the field notes and interviews with the instructors. For more information about the project, see authors (2008). The analysis of the video tapes used an Iterative Refinement Cycle (Lesh & Lehrer, 2000) model in which multiple interpretive cycles were used. The first interpretive cycle identified issues that pertained to general pedagogy and classroom culture. The second cycle identified issues that reflected the semiotic framework that framed this research investigation. The third cycle established explicit connections between the instructor's pedagogical decisions in order to make clear to the students the semiotic issues in their problem solving process. Finally, throughout the entire iterative viewing and interpretive process, analysis of other data sources helped inform the context and nature of the classroom discourse.

In our paper, we provide excerpts of the interpretive narrative, from two mathematics instructors of prospective elementary teachers. We show how Pat (a pseudonym) used semiotics to instruct his students about fractions, and then how Eliot (a pseudonym) could also use semiotics to help with instruction. Here, we illustrate the analysis for Pat, with a complete analysis of both instructors in the full paper.

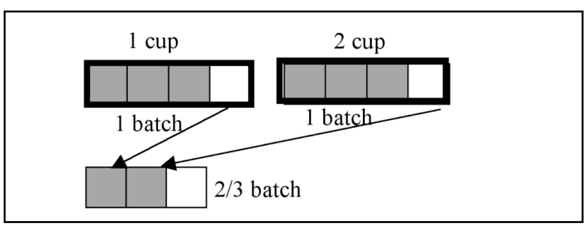
Results

Authors (2009) describe in detail how Pat taught division of fractions using a semiotic framework and how he explicitly taught semiotics prior to the instruction on fractions. Here, we focus on one segment of his teaching, his use of a word problem to teach partitive division:

> The Waffle Problem: *A batch of waffles requires ¾ of a cup of milk. You have two cups of milk. Exactly how many batches of waffles could you make?*
> (Class handout, 4/10/08)

After giving small groups time to work, Pat asked for solutions to the first problem. Students offered four different answers – 2, 2 ¼, 2 2/3, 2 3/8. The group that got 2 ¼ went to the board to explain their solution. One student said, "I knew that 1 cup of milk was four fourths," and then wrote out on the board 4/4 + 4/4 = 8/8 to represent the 2 cups of milk. The student continued saying, "I know ¾ a cup is batch so I took away 6/8 for two batches," while writing 4/4 + 4/4 = 8/8 – 6/8 = 2/8. The student concluded by saying that he had 2 batches so far, represented by the 6/8, and a ¼ leftover, simplified from the 2/8, giving a final answer of 2 ¼ batches. Another student quickly pointed out that 4/4 plus 4/4 was actually 8/4, not 8/8. Using that fact, Pat reworked the problem on the board writing 4/4 + 4/4 = 8/4, and 8/4 – 6/4 (for the two batches of waffles) = 2/4 = ½. The class then began to discuss the meaning of the symbol ½, whether it meant ½ a batch or ½ a cup. After some class discussion on how to interpret the symbol ½, a third student pointed out that the ½ left over was not ½ a BATCH, but rather was ½ a CUP and that ½ cup was the same as 2/3 of a batch. To illustrate his point, the student drew a pictorial representation of the problem (figure 1) trying to show how ½ cups of milk was equivalent to 2/3 batch of waffles.

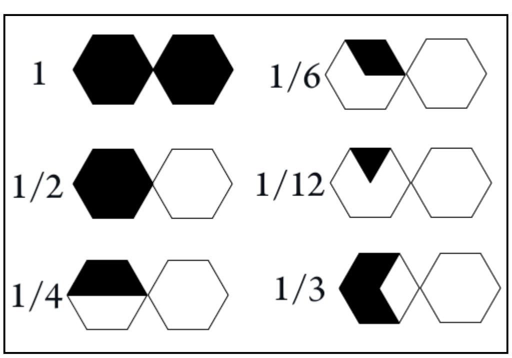
Figure 1: Representation of the waffle problem



After the students explained their answers, Pat explained how both a cup and a batch could be a whole. Pat stressed that the students needed to attend to the context: "technically the ½ is not wrong until you put a name to it. You say ½ a batch. That's not true, because it's half a cup." (from video on 4/10/08 and 4/15/08).

This is clearly a semiotic problem, where the signifier "1/2" had taken on two different meanings, depending on the signified, or concept, with which the students associated the signifier to construct their sign. One student incorrectly associated the ½ to 'batches' while another student correctly associated the ½ to 'cups.' In this case, the single signifier, ½, could only be correctly associated with one meaning, 'cup.' We describe this phenomenon with the phrase 'semiotic mismatch,' in which a signifier is incorrectly associated with a particular signified as determined by the context. As illustrated in this excerpt, by explicitly showing students how signs can mean different things depending on the context, the students gained insight in how to determine what the accurate signified, or intended meaning, was in the problem. In this class, the norm was for students to clearly identify and

Figure 2: The two-hexagon model



explain the meaning – the signified – for every sign, whether a number or a drawing.

In our second example, Eliot uses pattern blocks and drawings of those pattern blocks so that students can learn through hands-on experience. In the model she used, two hexagons were the whole, with other shapes representing different fractional parts as shown in Figure 2.

In this case, the physical block is the signifier, the quantity is the signified, and those two together create the sign. When the hexagon is drawn, then the drawing is the signifier, and the signified becomes more complicated because it represents both the quantity and the physical block. This added level of meaning can confuse students if they do not understand the semiotic chain at play. An example of this type of confusion occurs when the teacher or student refers to the shape rather than the actual number. Although it is appealing for a teacher to be able to say "how many rhombuses in a 3 hexagons?" rather than "what is 1 1/2 divided by 1/6?," the example does not clearly lead to understanding of fractions without explicit attention to the different meanings that are inherent in the words, pictures, and symbols. In the full paper, we illustrate Eliot's approach, and how a semiotic approach might have helped.

Significance of this Study: Overcoming Semiotic Confusion

In this paper, we illustrate how explicit attention to the semiotics of a mathematical problem can be used to help preservice math teachers develop a better understanding of both the mathematics of fractions and of the complexity of these ideas for their future students. We argue that not only is explicitness a way to help preservice teachers learn, but also it equips them to use (and decipher) tools they are given when they become teachers. Not every curriculum will use pattern blocks to teach fractions, but most K-8 curricula use one or more representations or manipulatives other than number symbols. Teachers need to be able to attach meaning to these representations and to deal with the complexity of these ideas for their students. We assert that the approach of stepping back a level and viewing the various representations as part of a systems of signs and signifiers can equip elementary teachers to apply their understanding across multiple and varied contexts. Whether this knowledge transfers to the elementary classroom is an area that needs further study. Nonetheless, the study of semiotics helps students to interrogate "how" things mean, not just what they mean, and to understand that a secondary system underpins the superficial representations of concepts with which they have become so familiar. Further, breaking down problems using semiotics makes explicit what we often think we are doing implicitly. Finally, this explicit decomposing of problems allows students to begin to see where their future students may encounter difficulties when trying to understand fractions.

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[i] Although many books talk about rational numbers rather than fractions, we use "fraction" since we are dealing exclusively with positive rational numbers in the form a/b.