Examining the Use of Reflection to Teach Fractions to Prospective Elementary Mathematics Teachers

Jessica Liu, Michigan State University
Esther Billings, Grand Valley State University
Andrea Francis, Michigan State University
Raven McCrory, Michigan State University

Purpose of the Study

In his book, *How We Think*, John Dewey argues that reflection is a powerful tool for gaining intentionality as a learner (1933). Others have written about how reflection can lead to increasing self awareness, tempering impulsive action, setting new teaching objectives, or bringing out new insights (Fendler, 2003). Reflection has been used explicitly as a tool in teacher mathematics education classes, but seemingly more often as an approach to methods than to the mathematical content (Cooney et al., 1999; Fuller & Bown, 1975; Mewborn, 1999). In this paper, we bring together reflection and mathematics content in a description of how one instructor incorporates reflection into a mathematics class for future teachers using a workshop model adapted from literacy instruction.

Theoretical Framework

Research suggests that teachers' mathematical knowledge can be an essential ingredient in effective mathematics instruction (Hill, et al, 2005; Ma, 1999; RAND Mathematics Study Panel, 2003). Mathematics classes for future elementary teachers provide one site for influencing what teachers know about mathematics. Reflection is one way to shape mathematical knowledge. In this study, our focus is on future teachers' learning to think about what mathematics they know and do not know, and on what they may expect from their future students.

While there are many conceptualizations of reflection, we assume that reflection includes both verbalizing and action (Ginsburg & Clift, 1990). From this perspective reflection involves more than just recollection or rationalization. Reflection forces students to verbalize the recollections and then act upon the rationalizations. Verbalizing during and after problem solving often leads to better understanding (Van Boxtel et al, 2000). Further, reflection often occurs in group situations. In what Elbers (2003) refers to as collective reflection, both "think[ing] about tools of argumentation" and the need to make convincing arguments are required (p. 92). In other words, students must account for their solutions to themselves and fellow students. Students know that they are supposed to argue for their ideas and, challenged by others' comments, to elaborate or improve them (Wells, 1993). In a sense, reflection in the form of verbalization creates an environment where the student becomes a researcher who must also rationalize and explain thoughts.

In this study, one teacher builds upon reflection work done in literacy by adapting a literacy workshop model (Cambourne, 1988) to instruct students in mathematical content and pedagogy. This workshop model provides a way of ensuring explicit time for reflection, as well as framing in-class lessons and homework. The workshop model includes five key components: *schema activation, focus, activity, reflection and extension* (see figure 1). *Schema activation*, building from Keene and Zimmerman's work (1997), involves students actively recalling and reflecting on what they currently know and making connections to past experiences and knowledge before beginning a new topic. The *focus* component sets expectations for the workshop; here students may be introduced to the theme of the lesson, a main question to explore, a mathematical strategy, children's mathematical thinking, or an article to read. The *activity* is the bulk of the workshop; students actively solve problems, collectively and individually exploring new ideas and connecting ideas. The teacher guides the students' problem-solving by asking questions and encouraging students to pose their own questions and construct new knowledge. Students may then choose activities to continue their exploration of the topic. After completing activities, students engage in *reflection* on what they have learned. Students collectively share their reflections, as well as individually record them. *Extensions* are also provided, either open ended or guided, giving students the opportunity to explore the topic in more depth or from a different perspective. It is important to note that the different components of the workshop model are rarely entirely separate from each other. An activity could act as a way to activate schemas, reflect, or even extend knowledge. The model is more of an explicit way of making sure that each of these components is present when preparing and presenting the lessons.

In the class described here, reflection realized through the workshop model was used to increase prospective teachers' mathematical content knowledge, and understanding of future student thinking. This is different than the use of reflection about teaching practices because of the focus on mathematics: there is *correct* mathematics content to learn. One content area that prospective teachers have trouble with is fractions (Graeber et al, 1989; Harel et al., 1994; Simon & Blume, 1994). Fractions have been defined in myriad ways. We use Lamon's (2007) suggestion that fractions are a subset of rational numbers, in that fractions are notational and are "non-negative rational numbers" (p. 635). A difficult topic for students to understand is the concept of the unit (eg., Isaak, 2008). Below we show how the instructor used reflection to help students learn about the concept of unit.

Method of Inquiry and Data Sources: A Case of One Teacher Using Reflection in a Prospective Elementary Teacher Mathematics Class

As part of a larger study that explores the mathematics content taught to undergraduate prospective elementary teachers, this paper focuses on one of seven instructors videotaped teaching fraction lessons. Video was supplemented by field notes taken during instruction and an interview with the instructor.[i] We chose a single instructor for this paper because of her unique approach, using the workshop model, to help her students be reflective as they learned mathematics. Over 15 hours of video was gathered for this instructor. Analysis of the video used an Iterative Refinement Cycle (Lesh & Lehrer, 2000) model with multiple interpretive cycles. The first cycle identified when and where the instructor explicitly explained reflective techniques for the students. The second cycle identified when the instructor explicitly used reflective techniques with her students. The third cycle was used to identify where the techniques were used implicitly in the classroom. The fourth cycle identified places where students were encouraged to use reflection when instructing their own students. Finally, throughout the entire process, other data sources, (field notes and interview) were used as supplements.

Below we provide excerpts of the interpretive narrative and discuss how Edie (a pseudonym), a mathematics professor teaching prospective elementary school teachers, used reflective techniques to teach about fractions.

Results

Edie teaches a combined methods and content class at a public university. The class is designed to explore the teaching and learning of number and operations (whole numbers, fractions, decimals, and number theory) in elementary school mathematics emphasizing the development of number sense and unitizing. Fieldwork for this course includes evaluating and tutoring elementary children. There were 11 students in the class, nine of whom were female. Students sat in assigned groups at three tables.

In this section of the paper we discuss how Edie used reflection as an instructional tool in the *schema activation* portion of the workshop model. In the full paper we will describe how reflection was used in the other elements of the workshop.

Edie began the fraction workshop by figuring out what students thought they knew about fractions. Edie helped the students activate their schema of fractions by asking the groups to develop a collective list of things they knew about fractions. In the groups, students were able to verbalize their understanding of fractions. One issue that the groups brought up during their collective reflection was how to represent the fraction. Many of the groups talked about using circles to represent fractions, but that there might be other ways to do it. By activating schemas, students personally saw their lack of knowledge about how to represent fractions and the need to learn more about representing fractions. The action that was needed was to then learn more about representing fractions.

Based on Edie's prior knowledge of previous years' students' lack of understanding about representing fractions and to further build upon their fractions schema, Edie had the prospective teachers solve the following problem from a grades 4-6 module on fractions.

> *Sub Problem: A class goes on a field trip. The first group has 4 people, and they get three sub sandwiches to share. The second group has 5 people and gets 4 subs. The third group has 8 people and gets 7 subs. The last group has 5 people and gets 3 subs. Which group gets the most food per person?* (Fosnot, 2007)

By having her students solve a problem, Edie provided a context for identifying additional types of understanding that her students could not verbalize when simply asked about what they knew about fractions. While the problem solving is part of the activity portion of the workshop, it also forces the students to reflect and activate their fraction schema as they must verbalize what they know about representing and comparing fractions in order to solve the problem.

To get the students to make explicit how they solved the problem, Edie first had each of the groups create a poster to represent each of the situations. Edie asked the students to draw it in such a way that their own future students would be able to understand how to compare fractions. In the next class period, Edie had students discuss the posters they made to represent the submarine problem. Edie asked, "What kind of strategy and representation did your classmates make in order to answer this question?" and "what is the underlying meaning of the fraction?"

The activity, the action of presenting the problem to each other, was explicitly designed to have students recall and rationalize what they did in the previous class period so that Edie could further build upon that existing knowledge. In this way, the activity was also an example of schema activation. Through a whole class discussion of each group's posters, the students engaged in collective reflection by sharing and analyzing each other's solutions and identifying what they did not understand about fraction representation. For example, to figure out how much the first group in the submarine problem gets, one of the groups represented the problem by putting the three subs together into one large rectangle, divided each sub into 4, and said that each person got 3/12 of a sub, which was an incorrect answer (Figure 2). After Edie asked the group how they got 3/12 of a sub, the group explained their representation in a way that suggested they got ¾ of sub (the correct answer). One of the students in the group responded, "I don't know what I did [the first time]." The class discussion had influenced the way this student solved the problem. She had listened to and understood the other groups' explanations and used them to correct her own representation. However, the student realized she could not argue for how the problem was originally solved because she did not understand where she made a mistake in her own reasoning; she did not comprehend the importance of a fractional unit Ultimately, the vocalization of how they solved the problem – activating their schema of fractions -- allowed the students to see that they had made a mistake about the unit and whole (one sub versus three subs) involved in solving the problem. Further, the students actually figured out what the correct answer was to the problem (3/4) (see the dialogue in figure 3 for the complete transcription of this part of the lesson).

Significance of the Study

In this paper we have shown how one instructor used the instructional tool of reflection, made more explicit in the workshop model, to help students understand what they do and do not understand about units in fractions. To our knowledge this is the first time the workshop model, modified from instructional approaches in literacy, has been explicitly used in a mathematics class for prospective elementary teachers. Further, this is an example of the use of reflection as an instructional tool to teach students to figure out what mathematics content they do not fully understood. We ultimately hope that this approach will encourage prospective teachers to take responsibility for their own learning by reflecting on what they do and do not know about mathematics topics.

References

Cambourne, B. (1988). *The Whole Story: Natural learning and the acquisition of literacy*. Auckland, New Zealand, Ashton Scholastic.

Cooney, T., Shealy, B., & Arnold, B. (1998). Conceptualizing belief structures of preservice secondary mathematics teachers. *Journal of Research in Mathematics Education, 29(3),* 306-333.

Dewey, J. (1933). *How we think: A restatement of the relation of reflective thinking to the educative process*. Boston: Heath.

Elbers, E. (2003). Classroom interaction as reflection: Learning and Teaching Mathematics in a Community of Inquiry. *Educational Studies in Mathematics* (p.77-99). Netherlands: Kluwer Academic Publishers.

Fendler, L. (2003). Teacher Reflection in a Hall of Mirrors: Historical Influences and Political Reverberations. *Educational Researcher* (p.16-25).

Fosnot, C. T. (2007). Day One: The Field Trip (pp. 13-18). Contexts for Learning Mathematics Grades 4-6": Field Trips and Fund-Raisers: Introducing Fractions. Portsmouth, NH: Firsthand Heinemann.

Fuller, F. F., & Bown, O. H. (1975). Becoming a teacher. In K. Ryan (Ed.), *Teacher education: Seventy-fourth yearbook of the National Society for the Study of Education,* (Part II, pp. 25-52). Chicago: National Society for the Study of Education.

Ginsburg, M. B., & Clift, R. T. (1990). The hidden curriculum of preservice teacher education.In W. R. Houston Macmillan. (Ed.), *Handbook of research on teacher education* (p. 450-465). New York: Macmillan.

Graeber, A., Tirosh, D., & Glover, R. (1989). Preservice teachers' misconceptions in sovivng verbal problems in multiplication and division. *Journal for Research in Mathematics Education, 20(1),* 95-102.

Harel, G, Behr, M., Post, L., & Lesh, R. (1994). The impact of number type on the s solution of multiplication and division problems. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 363-384). New York: State University of New York Press.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Izsak, A. (2008). Mathematical knowledge for fraction multiplication. *Cognition and Instruction, 26*, 95-143.

Keene, E.O., Zimmerman, S. Mosaic of Thought: Teaching Comprehension in a Reader's Workshop. Portsmouth: Heinemann. 1997.

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. In *Handbook of Research on Mathematics Education* (pp. 629-667).

Lesh, R., & Lehrer, R. (2000). Iterative refinement cycles for videotape analysis of conceptual change. In A. E. Kelly & R. A. Lesh (Eds.), Handbook of research design in mathematics and science education (pp. 992). Mahwah, NJ: Lawrence Erlbaum Associates.

Ma, L. (1999). *Knowing and teaching elementary mathematics. Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum.

Mewborn, D. S. (1999). Reflective thinking among preservice elementary mathematics tachers. Journal for Research in Mathematics Education, 30(3), 316-341.

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and Standards of School Mathematics*. Reston, VA: NCTM.

RAND Mathematics Study Panel (2003). *Mathematical proficiency for all students: Towards a strategic development program in mathematics education*. Santa Monica, CA: RAND Corporation MR-1643-OERI.

Simon, M.A., & Blume, G.W. (1994). Building and understanding multiplicative relationships: A study of prospective elementary teachers. *Journal for Research in Mathematics Education, 25(5)*, 472-494.

Van Boxtel, C., Van der Linden, J., & and Kanselaar, G. (2000). *Collaborative Learning Tasks and the Elaboration of Conceptual Knowledge*.

Wells, G. (1993)Reevaluating the IRF sequence: A proposal for the articulation of theories of activity and discourse for the analysis of teaching and learning in the classroom. *Linguistics and Education, 5*, 1-37.

Figure 1: Adapted Workshop Model from Cambourne (1988)

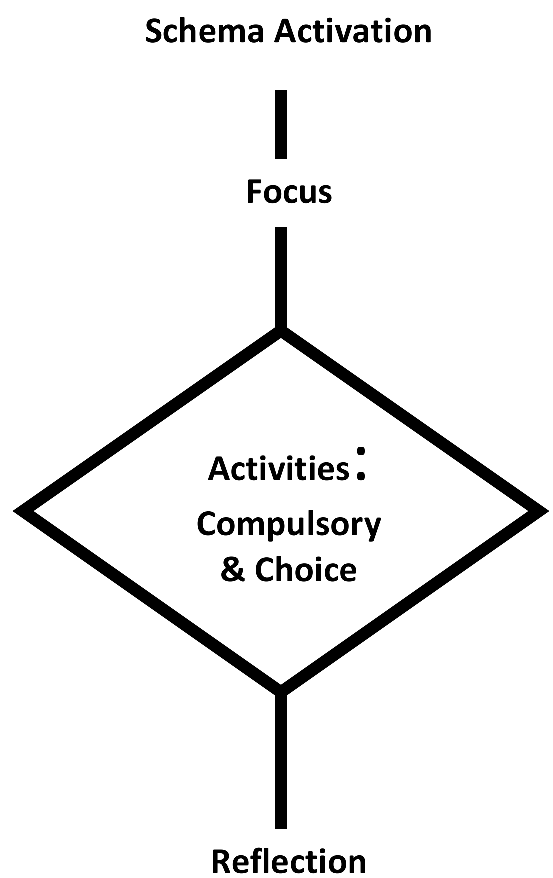

Figure 2: Group three's use of an array to represent sandwiches and people

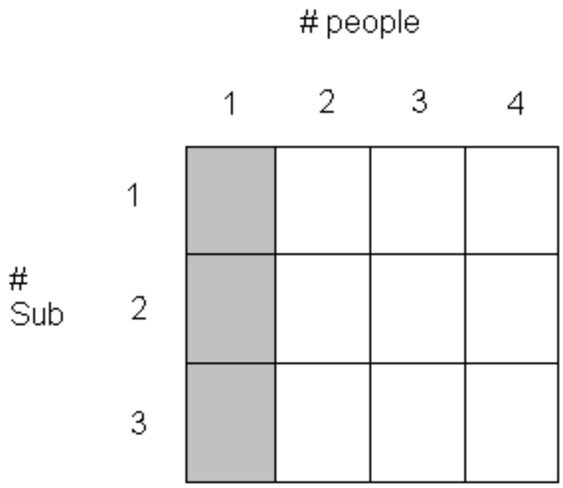

Figure 3: Dialogue of students overcoming an incorrect answer

Edie: now, what your group did … [draws a big square] what does this big rectangle represent?
Student 1: well, we divided it up into sub-squares, then…
Edie: [divides square into 3 rectangles] so here are your three subs, okay, [labels 1,2,3 vertically.
Student 1: and you have to divide that [the top side] by 4. So in the first column, you have ¼
Edie: one fourth of what?
Student 1: of a sub
Edie: so you've got ¼ of a sub [shades ¼], another fourth of a sub [shades fourth], another fourth of a sub [shades fourth]. So how do you know that's three fourths?
Student 2: we added them up.
Edie: at one point, you said there were 3/12s sub. Now, did each person get 3/12 of a sub?
Student 1: I don't know what we did.
Student 3 : all the subs together.
Edie: all the subs together. If we had divided our 3 subs, right, into 4 pieces, right? We've got 12 parts, right? Three of the parts. So we've got 3/12 of a sub or 3/12 of what?
Student 4: 3 subs
Edie: [nods] 3/12, right, of 3 subs. Now, 3/12s of 3 subs, what – do you know what fraction that is?
Students: ¼

---

[i] For additional information see [authors, 2008]; Web site http://meet.educ.msu.edu/